Guidelines for Solving First-Order Differential Equations

Form of Equation	Method	Solution
$\frac{dy}{dx}=\frac{g(x)}{f(y)}$	Separate the variables	$\int f(y)dy=\int g(x)dx$
$\frac{dy}{dx}=f(\frac{y}{x})$	Homogeneous-Use a change of variable $v=\frac{y}{x}$ and $\frac{dy}{dx}=v+x\frac{dv}{dx}$	$\int\frac{dv}{f(v)-v}=\int\frac{dx}{x}$
$\frac{dy}{dx}+p(x)y=q(x)$	Use the integrating factor $I(x)=e^{\int p(x)dx}$	$y=\frac{1}{I(x)}\int I(x)q(x)dx$
$\frac{dy}{dx}=$ ____ (Isolate $\frac{dy}{dx}$)	Use Euler's Formula for numerical Evaluation $y_{n+1}=y_n+h\times f(x_n,y_n)$ $x_{n+1}=x_n+h$	(x_f,y_f)

1. Separable Equations:

$M(x)dx+N(y)dy=0$

Example 1)

$9dx-x\sqrt{x^2-9}dy=0$

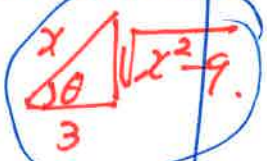
$9dx=x\sqrt{x^2-9}dy$

$\frac{9dx}{x\sqrt{x^2-9}}=dy$

$\int dy=\int\frac{9dx}{x\sqrt{x^2-9}}$

$x=3\sec\theta$

$dx=3\sec\theta\tan\theta d\theta$



$3\tan\theta=\sqrt{x^2-9}$

$y=\int\frac{9\cdot 3\cdot\sec\theta\cdot\tan\theta d\theta}{3\sec\theta\cdot 3\tan\theta}$

$y=\int 3d\theta=3\theta+C$

$y=3\,\text{arcsec}(\frac{x}{3})+C$

Example 2)

$xy\frac{dy}{dx}=x^2+y^2+x^2y^2+1$

$xy\frac{dy}{dx}=x^2(1+y^2)+1(1+y^2)$

$xy\cdot\frac{dy}{dx}=(1+y^2)(x^2+1)$

$\int\frac{ydy}{(1+y^2)}=\int(\frac{x^2+1}{x})dx$

$u=y^2+1$

$\frac{1}{2}du=y\cdot dy$

$\frac{1}{2}\ln(y^2+1)=\frac{1}{2}x^2+\ln x+C$