Exit slip #2 Solution.

#1

$f(x) = \sin(x) \Rightarrow \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

$f'(x) = \cos x \Rightarrow \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$f''(x) = -\sin x \Rightarrow -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$

$\Rightarrow M_3(x) = \frac{1}{2} + \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right) + \left(\frac{-1}{4}\right)\left(x - \frac{\pi}{6}\right)^2 + R_2(x)$

#2. $f(x) = \ln\left(\frac{1+x}{1-x}\right)$

$= \ln(1+x) - \ln(1+x) = \left[x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \cdots\right]$

$- \left[-x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} \cdots\right]$

$M_5 = 2x + \frac{2}{3}x^3 + \frac{2}{5}x^5 + \frac{2}{7}x^7 + R_7(x)$

$\frac{1+x}{1-x} = 3 \Rightarrow x = \frac{1}{2}$

$M_5\left(\frac{1}{2}\right) = (2)\left(\frac{1}{2}\right) + \frac{2}{3}\left(\frac{1}{2}\right)^3 + \frac{2}{5}\left(\frac{1}{2}\right)^5 + \frac{2}{7}\left(\frac{1}{2}\right)^7 \approx 1.09807$ ~~1.0032~~

$R_{5\,MAX} = \frac{2}{9}\left(\frac{1}{2}\right)^9 + \frac{2}{9}\left(\frac{1}{2}\right)^{11} + \frac{2}{9}\left(\frac{1}{2}\right)^{13} \cdots$

$= \left(\frac{2}{9}\right)\left(\frac{1}{2}\right)^9 \cdot \frac{1}{1-\frac{1}{4}} \approx 0.0005787$

$1.09807 - 0.0005787 \le \ln 3 \le 1.0\ldots + 0.0005787$

$1.09749 \le \ln 3 \le 1.09864.$

#3. $\int_0^x \frac{1}{1+t^2}\,dt = \int [1 - t^2 + t^4 - t^6 + t^8 \ldots]\,dt$

$\Rightarrow \tan^{-1}x - \tan^{-1}(0) = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 \ldots$

$x = 1 \Rightarrow \frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7}$

$\pi = 4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7}\right) \approx 2.90$

OR $x = \frac{1}{\sqrt{3}} \Rightarrow \frac{\pi}{6} = \frac{1}{\sqrt{3}} - \frac{1}{3}\left(\frac{1}{\sqrt{3}}\right)^3 + \frac{1}{5}\left(\frac{1}{\sqrt{3}}\right)^5 - \frac{1}{7}\left(\frac{1}{\sqrt{3}}\right)^7$

$\pi = 6 \cdot (0.945625) \approx \boxed{3.14}$

#4. $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + R_7(x)$

$x = 1.5 = \frac{3}{2}$

$\sin(1.5) = \left(\frac{3}{2}\right) - \frac{\left(\frac{3}{2}\right)^3}{3!} + \frac{\left(\frac{3}{2}\right)^5}{5!} - \frac{\left(\frac{3}{2}\right)^7}{7!} \approx 0.997391$

$R_7(1.5) = \frac{(\sin x)^9(z)}{9!}\left(\frac{3}{2}\right)^9$

≈ 0.000106

$(\sin x)^9 = \cos x$

$0 \le z \le 1.5$

$\cos(0) = 1$ (max)

$0.997391 - 0.000106 \le \sin(1.5) \le 0.997391 + 0.000106$

$0.997285 \le \sin(1.5) \le 0.997497$

$\boxed{0.9973 \le \sin(1.5) \le 0.9975}$