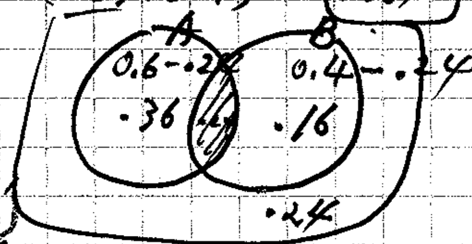
Probability / Stat. Review

#1.

a. $A \cap B = P(A) \cdot P(B) = (0.6)(0.4) = \boxed{.24}$



b. $A \cup B = \boxed{.76}$

c. $A / B' = \dfrac{\overset{P(A \cap B)}{.36}}{(1-0.4)} = \boxed{0.6}$

#2 $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = 0.75$ $P(B) = 0.4$ $P(A) =$ [illegible]

$P(A \cap B) = (0.75)(0.4) = 0.3$

$P(B|A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{0.3}{0.45} = 0.\bar{6}$

#3. (a) $2 \times 4! \times 1 = 48$

4 _ 4! 3 3 (b) $4 \times 4! \times 3 = 288$

(c) ${}_4C_2$ $6! - 5!2! = 480$.

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#3. Marian problem

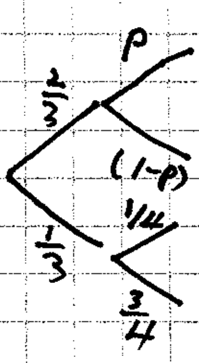
mean: $(10)(0.4) = 4$

$\sigma^2 = n \cdot p \cdot q = 2.4$

$\sigma = \sqrt{2.4}$

#4. $P(x|y') = \frac{1}{2}$.

$\dfrac{P(x \cap y')}{P(y')} = \dfrac{\frac{2}{3}(1-p)}{\frac{2}{3}(1-p) + (\frac{1}{3} \times \frac{3}{4})} = \frac{1}{2}$.



Solve for $p = \frac{5}{8}$.

y: on time

y': not on time.

#5

a. $p(N \le 4) = 0.1 + 0.3 + 0.25 + 0.2 = 0.85$

b. $p(N \ge 3 / N \le 4) = \dfrac{p(N=3,4)}{p(N \le 4)} = \dfrac{0.25+0.2}{0.85}$

$= 0.5294$

c. $E(N) = (0.1) + 2(0.3) + 3(0.25) + 4(0.2)$
$+ 5(0.15) = 3$

d. $E(N^2) = (1)^2(0.1) + (2)^2(0.3) + (3)^2(0.25) + (4)^2$[illegible]
$+ (5)^2(0.15)$

Variance $= (E(N))^2 - E(N^2))$
$= \sqrt{1.5}$

e. $E(2N+1) = 2E(N) + 1 = 7$
$2(3) + 1$

e $Var(3N+2) = 3^2 Var(N) = 3^2 \sqrt{1.5} = 13.5$

#6 $p(\text{defected}) = 0.1$ X

$p(X \ge 1) = 1 - p(X=0)$
$= 1 - (0.1)^0 (0.9)^8 = 0.5695$

#7. a. $E(x) = \left(\frac{2}{5}\right)(\cancel{50}\ 10) = 20$

b. $\sigma^2 = n \cdot p \cdot (1-p) = (\cancel{50}\ \cancel{10}\ 2)\left(\frac{2}{5}\right)\left(\frac{3}{5}\right) = 12$

$\sigma = \sqrt{12} = 2\sqrt{3}$.

#8. Undersized = x

$p(x) = \frac{3}{16}$ $\qquad 1 - \frac{{}_3C_0 \, {}_{13}C_3}{{}_{16}C_3} \qquad 1 - \frac{286}{560}$

$$p(x > 1) = 1 - p(x=0) - p(x=1) \qquad - \frac{{}_3C_1 \, {}_{13}C_2}{{}_{16}C_3} \qquad - \frac{117}{280}$$

$$= 1 - \frac{286}{560} - \frac{117}{280} = \boxed{\frac{1}{14}}$$

#9 $\quad y = n \cdot t = (2.1)(1) = 2.1 \qquad$ poisson (y, x)

a) $p(x \geq 3) = 1 - p(x=0) - p(x=1) - p(x=2)$

$1 -$ poissoncdf $(2.1, 2) = 1 - 0.6496$

$$1 - \frac{e^{-2.1} \cdot (2.1)^0}{0!} - \frac{e^{-2.1}(2.1)^1}{1!} - \frac{e^{-2.1}(2.1)^2}{2!} = .3504$$

b) Binomial $\qquad$ binomcdf (n, p, x)

$p(x \geq 3) \Rightarrow 1 -$ binomcdf $(6, .3504, 2)$

$$= 1 - \underset{(1)}{{}_6C_0} (.3504)^0 \underset{.6496}{(1 - 0.3504)^6}$$

$$- \underset{(6)}{{}_6C_1} (.3504)^1 (1 - 0.3506)^5$$

$$- \underset{(15)}{{}_6C_2} (.3504)^2 (1 - 0.3\cancel{9}6)^4$$

$\approx .353722$ $\quad$ (~~.6817~~)

LEVEL 3

1. $X \sim N(35, 6.25)$ **i.** $p(X \geq 40) = p(Z \geq 2) = 0.0228$ **ii.** $p(X \geq c) = 0.1 \therefore \frac{c-35}{2.5} = \Phi^{-1}(0.9)$

$\therefore c = 35 + 2.5 \times \Phi^{-1}(0.9) = 38.20$

2. $p(X > 4) = p\left(Z > \frac{4-a}{b}\right) = 0.15 \Leftrightarrow \frac{4-a}{b} = \Phi^{-1}(0.85) \therefore (4-a) = b \times \Phi^{-1}(0.85)$ —(1)

$p(X \leq 3) = p\left(Z \leq \frac{3-a}{b}\right) = 0.7 \Leftrightarrow \frac{3-a}{b} = \Phi^{-1}(0.70) \therefore (3-a) = b \times \Phi^{-1}(0.70)$ —(2)

(1) − (2): $1 = b(\Phi^{-1}(0.85) - \Phi^{-1}(0.70)) \therefore b = \frac{1}{(\Phi^{-1}(0.85) - \Phi^{-1}(0.70))} = 1.953$

Sub into (1): $a = 4 - \frac{\Phi^{-1}(0.85)}{(\Phi^{-1}(0.85) - \Phi^{-1}(0.70))} = 1.976$

3. a. i. $p\left(\frac{1}{2}X + 4 > 16\right) = p(X > 24) = p\left(Z > \frac{24-20}{3}\right) = p\left(Z > \frac{4}{3}\right) = 0.0912$

ii. $p(2|X| - 5 < 45) = p(|X| < 25) = p(-25 < X < 25) = p(-15 < Z < 1.666) = 0.9522$

b. $p(X < 2a+1) = 0.3 \Rightarrow p\left(Z < \frac{2a-19}{3}\right) = 0.3 \therefore \frac{2a-19}{3} = \Phi^{-1}(0.30)$ $2a = 19 + 3 \times \Phi^{-1}(0.30)$

Therefore, $a = 8.71$ **c.** $p(X > 2a | X > 2a-1) = \frac{p(X > 2a)}{p(X > 2a-1)} = \frac{p(X > 19 + 3 \times \Phi^{-1}(0.30))}{p(X > 18 + 3 \times \Phi^{-1}(0.30))}$

$= \frac{0.80448}{0.88318} = 0.9109$ **d.** $p(X^3 - 18X^2 < 0) = p(X^2(X-18) < 0) = p(X < 18) = p\left(Z < -\frac{2}{3}\right) = 0.2525$

4. $\frac{2a-a}{\sqrt{b}} = \Phi^{-1}(0.85) \Rightarrow a = \sqrt{b} \times \Phi^{-1}(0.85)$ —(1)

$\frac{1-a}{\sqrt{b}} = \Phi^{-1}(0.65) \Rightarrow 1-a = \sqrt{b} \times \Phi^{-1}(0.65)$ —(2)

(1) + (2): $1 = \sqrt{b}(\Phi^{-1}(0.85) + \Phi^{-1}(0.65)) \therefore \sqrt{b} = \frac{1}{(\Phi^{-1}(0.85) + \Phi^{-1}(0.65))}$

$\Rightarrow b = \frac{1}{(\Phi^{-1}(0.85) + \Phi^{-1}(0.65))^2}$. Therefore, $b = 0.4947$ Sub into (1): $a = 0.729$

5. $X \sim N(50, 1.25^2)$, $p(49 < X < 52) = p(-0.8 < Z < 1.6) = 0.7333$. That is, 73.33%

6. $p(X < 2) = 0.6 \therefore p\left(\frac{X-x}{\sqrt{y}} < \frac{2-x}{\sqrt{y}}\right) = 0.6 \therefore \frac{2-x}{\sqrt{y}} = \Phi^{-1}(0.6) \Leftrightarrow 2-x = \sqrt{y}\Phi^{-1}(0.6)$ –(1)

Also, $p(X > 3) = 0.2 \Rightarrow p(X \leq 3) = 0.8 \therefore \frac{3-x}{\sqrt{y}} = \Phi^{-1}(0.8) \Leftrightarrow 3-x = \sqrt{y}\Phi^{-1}(0.8)$ –(2)

Solving, (2) − (1): $1 = \sqrt{y}[\Phi^{-1}(0.8) - \Phi^{-1}(0.6)] \therefore y = \left(\frac{1}{\Phi^{-1}(0.8) - \Phi^{-1}(0.6)}\right)^2 = (1.6998)^2 = 2.8896$

Sub. into (1): $x = 2 - 1.6998\Phi^{-1}(0.6) = 1.5693$. $\therefore p(X > 1) = 0.6312$.

7. $\frac{50-\mu}{\sigma} = \Phi^{-1}(0.5) \Leftrightarrow 50-\mu = \sigma\Phi^{-1}(0.5)$ –(1) & $\frac{80-\mu}{\sigma} = \Phi^{-1}(0.9) \Leftrightarrow 80-\mu = \sigma\Phi^{-1}(0.9)$ –(2)

(2) − (1): $30 = \sigma[\Phi^{-1}(0.9) - \Phi^{-1}(0.5)] \therefore \sigma = \frac{30}{\Phi^{-1}(0.9) - \Phi^{-1}(0.5)} = 10.251485\ldots$

Sub. into (1): $\mu = 50 - \frac{30}{\Phi^{-1}(0.9) - \Phi^{-1}(0.5)} \times \Phi^{-1}(0.5) = 66.8622$.

$\therefore p(X > 60) =$ **normalcdf(60,1000000,66.8622,10.2515)** $= 0.7484$

8. $T \sim N(12, 0.02^2)$ **a. i.** $p(T > 12.05) =$ **normalcdf(12.05,1000000,12,0.02)** $= 0.0062$

ii. $p(T < 11.96) =$ **normalcdf(−1000000,11.96,12,0.02)** $= 0.0228$

b. $p(T > 12.05 | T > 12.02) = \frac{p(T > 12.05)}{p(T > 12.02)} = \frac{0.006209\ldots}{0.158655\ldots} = 0.0391$

c. $p = p(T < 11.96) = 0.0228$, $N \sim Bin(10, 0.0228)$. $p(N \leq 2) =$ **binomcdf(10,0.0228,2)** $= 0.9987$

Name: ____________________

1) Derive the Poisson distribution $p(x) = \dfrac{e^{-\mu}\mu^x}{x!}$ from the Binomial distribution

$p(x) = {}_nC_x p^x (1-p)^{n-x}$ as $n \to \infty$.

Binomial

$$p(x) = \left[{}_nC_x p^x (1-p)^{n-x} \right] = \left[\frac{n!}{(n-x)!\,x!} p^x (1-p)^{n-x} \right] \quad E(x) = n \cdot p = \mu \Rightarrow p = \frac{\mu}{n}$$

sub.

$$p(x) = \frac{n!}{(n-x)!\,x!} \left(\frac{\mu^x}{n^x} \right) \left(1 - \frac{\mu}{n} \right)^{n-x}$$

$$= \frac{n!}{(n-x)!\,x!} \left(\frac{\mu^x}{n^x} \right) \left(\frac{n-\mu}{n} \right)^n \left(\frac{n}{n-\mu} \right)^x$$

Poisson $(n \to \infty)$

$$= \frac{n(n-1)(n-2)\cdots\cancel{(n-x)!}}{\cancel{(n-x)!}\; x!} \cdot \left(\frac{\mu^x}{\cancel{n^x}} \right) \left(\frac{(n-\mu)^n}{n^n} \right) \left(\frac{\cancel{n^x}}{(n-\mu)^x} \right) = \frac{n(n-1)(n-2)\cdots(n-(x+1))}{x!\;(n-\mu)^x} \mu^x \cdot \left(1 - \frac{\mu}{n}\right)^n$$

$$\lim_{n \to \infty} \frac{\overbrace{n(n-1)(n-2)\cdots(n(x+1))}^{x}}{(n-\mu)^x\; x!} \left(1 - \frac{\mu}{n}\right)^n (\mu^x) = \boxed{\frac{e^{-\mu} \cdot \mu^x}{x!}}$$

$\to 1$ $\quad + \dfrac{e^{-\mu} \cdot \mu^x}{x!}$

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2) Derive the $E(x) = \mu$ for the Poisson Distribution. $\quad \mu = \lambda t$

$$\text{var}(x) = E(x^2) - (E(x))^2 = \mu^2 + \mu - \mu^2 = \mu$$

$E(x) = n \cdot p$ (for Binomial)

$$\Rightarrow E(x) = \sum_{x=0}^{\infty} x \cdot \frac{e^{-\mu} \cdot \mu^x}{x!} \qquad (x-1=y) \Rightarrow (x=y+1)$$

$$= \sum_{y=1}^{\infty} \frac{(y+1) \cdot e^{-\mu} \cdot \mu^{(y+1)}}{(y+1)!\; y!} = e^{-\mu} \cdot \mu \cdot \sum_{y=1}^{\infty} \frac{\mu^y}{y!} = e^{-\mu} \cdot \mu \cdot e^{\mu} = \boxed{\mu}$$

$$\left(\sum_{y=1}^{\infty} \frac{\mu^y}{y!} = \frac{\mu}{1} + \frac{\mu^2}{1 \cdot 2} + \frac{\mu^3}{1 \cdot 2 \cdot 3} + \frac{\mu^4}{1 \cdot 2 \cdot 3 \cdot 4} \cdots = e^{\mu} \right)$$

$$\Rightarrow E(x^2) = \sum_{x=0}^{\infty} \frac{x^2 \cdot e^{-\mu} \cdot \mu^x}{x!} = \sum_{y=1}^{\infty} \frac{(y+1)^2 (e^{-\mu}) \cdot \mu^{y+1}}{(y+1)!\; y!}$$

$$= \sum_{y=1}^{\infty} \left(\frac{y+1}{y!} \right) (e^{-\mu}) \cdot \mu^y \cdot \mu = e^{-\mu} \cdot \mu \left[\sum \frac{y \cdot \mu^y}{y!} + \sum \frac{\mu^y}{y!} \right]$$

$$= \mu \sum \frac{y \cdot e^{-\mu} \cdot \mu^y}{y!} + \mu \cdot e^{-\mu} \cdot e^{\mu} = \mu^2 + \mu$$