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Probability Distribution (Exit Slip): IB Exam questions : Work in graph paper

1. *[Maximum mark: 5]*

The probability density function of the random variable X is defined as

$$f(x) = \begin{cases} \sin x, & 0 \le x \le \dfrac{\pi}{2} \\ 0, & \text{otherwise.} \end{cases}$$

Find $\mathrm{E}(X)$.

[Maximum mark: 5]

2. Emily walks to school every day. The length of time this takes can be modelled by a normal distribution with a mean of 11 minutes and a standard deviation of 3 minutes. She is late if her journey takes more than 15 minutes.

(a) Find the probability she is late next Monday. *[2 marks]*

(b) Find the probability she is late at least once during the next week (Monday to Friday). *[3 marks]*

[Maximum mark: 7]

3. A ferry carries cars across a river. There is a fixed time of T minutes between crossings. The arrival of cars at the crossing can be assumed to follow a Poisson distribution with a mean of one car every four minutes. Let X denote the number of cars that arrive in T minutes.

(a) Find T, to the nearest minute, if $P(X \le 3) = 0.6$. *[3 marks]*

It is now decided that the time between crossings, T, will be 10 minutes. The ferry can carry a maximum of three cars on each trip.

(b) One day all the cars waiting at 13:00 get on the ferry. Find the probability that all the cars that arrive in the next 20 minutes will get on either the 13:10 or the 13:20 ferry. *[4 marks]*

4. *[Maximum mark: 5]*

The weights, in kg, of one-year-old bear cubs are modelled by a normal distribution with mean μ and standard deviation σ.

(a) Given that the upper quartile weight is 21.3 kg and the lower quartile weight is 17.1 kg, calculate the value of μ and the value of σ. *[4]*

A random sample of 100 of these bear cubs is selected.

(b) Find the [illegible]

[Maximum mark: 6]

5. The continuous variable X has probability density function

$$f(x)=\begin{cases}12x^2(1-x), & 0\le x\le 1\\ 0, & \text{otherwise.}\end{cases}$$

(a) Determine $\mathrm{E}(X)$. *[3 marks]*

(b) Determine the mode of X. *[3 marks]*

[Maximum mark: 17]

6. The weights, in kg, of male birds of a certain species are modelled by a normal distribution with mean μ and standard deviation σ.

(a) Given that 70 % of the birds weigh more than 2.1 kg and 25 % of the birds weigh more than 2.5 kg, calculate the value of μ and the value of σ. *[4 marks]*

(b) A random sample of ten of these birds is obtained. Let X denote the number of birds in the sample weighing more than 2.5 kg.

(i) Calculate $\mathrm{E}(X)$.

(ii) Calculate the probability that exactly five of these birds weigh more than 2.5 kg.

(iii) Determine the most likely value of X. *[5 marks]*

(c) The number of eggs, Y, laid by female birds of this species during the nesting season is modelled by a Poisson distribution with mean λ. You are given that $\mathrm{P}(Y\ge 2)=0.80085$, correct to 5 decimal places.

(i) Determine the value of λ.

(ii) Calculate the probability that two randomly chosen birds lay a total of two eggs between them.

(iii) Given that the two birds lay a total of two eggs between them, calculate the probability that they each lay one egg. *[8 marks]*

[Maximum mark: 5]

7. Flowering plants are randomly distributed around a field according to a Poisson distribution with mean μ. Students find that they are twice as likely to find exactly ten flowering plants as to find exactly nine flowering plants in a square metre of field. Calculate the expected number of flowering plants in a square metre of field.

8. *[Maximum mark: 20]*

The probability density function of the random variable X is given by

$$f(x) = \begin{cases} \dfrac{k}{\sqrt{4-x^2}}, & \text{for } 0 \le x \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the value of the constant k. *[5 marks]*

(b) Show that $\mathrm{E}(X) = \dfrac{6(2-\sqrt{3})}{\pi}$. *[7 marks]*

(c) Determine whether the median of X is less than $\dfrac{1}{2}$ or greater than $\dfrac{1}{2}$. *[8 marks]*

[Maximum mark: 6]

9. The random variable X has a Poisson distribution with mean 4. Calculate

(a) $\mathrm{P}(3 \le X \le 5)$; *[2 marks]*

(b) $\mathrm{P}(X \ge 3)$; *[2 marks]*

(c) $\mathrm{P}(3 \le X \le 5 \mid X \ge 3)$. *[2 marks]*

10. *[Maximum mark: 6]*

The speeds of cars at a certain point on a straight road are normally distributed with mean μ and standard deviation σ. 15 % of the cars travelled at speeds greater than $90\ \mathrm{km\,h^{-1}}$ and 12 % of them at speeds less than $40\ \mathrm{km\,h^{-1}}$. Find μ and σ.

[Maximum mark: 6]

11. In a factory producing glasses, the weights of glasses are known to have a mean of 160 grams. It is also known that the interquartile range of the weights of glasses is 28 grams. Assuming the weights of glasses to be normally distributed, find the standard deviation of the weights of glasses.

12. *[Maximum mark: 22]*

A ski resort finds that the mean number of accidents on any given weekday (Monday to Friday) is 2.2. The number of accidents can be modelled by a Poisson distribution.

(a) Find the probability that in a certain week (Monday to Friday only)

(i) there are fewer than 12 accidents;

(ii) there are more than 8 accidents, given that there are fewer than 12 accidents. *[6 marks]*

Due to the increased usage, it is found that the probability of more than 3 accidents in a day at the weekend (Saturday and Sunday) is 0.24.

(b) Assuming a Poisson model,

(i) calculate the mean number of accidents per day at the weekend (Saturday and Sunday);

(ii) calculate the probability that, in the four weekends in February, there will be more than 5 accidents during at least two of the weekends. *[10 marks]*

It is found that 20 % of skiers having accidents are at least 25 years of age and 40 % are under 18 years of age.

(c) Assuming that the ages of skiers having accidents are normally distributed, find the mean age of skiers having accidents. *[6 marks]*

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