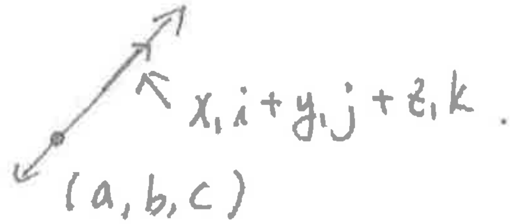
Equation of lines and planes.

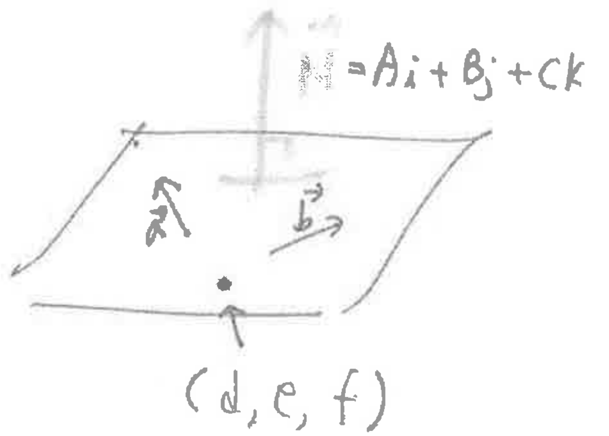
① Lines.



$$\boxed{\frac{x-a}{x_1} = \frac{y-b}{y_1} = \frac{z-c}{z_1}}$$

$$\hookrightarrow \; x = a + x_1 t$$
$$y = b + y_1 t$$
$$z = c + z_1 t.$$

② planes



$$\vec{r} = \begin{pmatrix} d \\ e \\ f \end{pmatrix} + \lambda \left(\vec{a} \right) + t \left(\vec{b} \right).$$

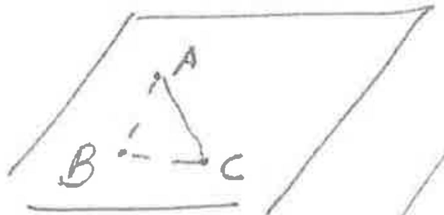
$$\Rightarrow \; Ax + By + Cz = D. \qquad \Rightarrow \; \frac{\vec{r} \cdot \vec{n}}{|n|} = \frac{\vec{a} \cdot \vec{n}}{|\vec{n}|} = d$$

$\vec{N}$ (normal vector to the plane)

$$\vec{N} = \vec{a} \times \vec{b}$$

Example) $A(1,2,3)$ $B(1,0,5)$ $C(2,-1,4)$

a)



$$\vec{AB} = \begin{pmatrix} 1-1 \\ 0-2 \\ 5-3 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \\ 2 \end{pmatrix}$$

$$\vec{AC} = \begin{pmatrix} 2-1 \\ -1-2 \\ 4-3 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}$$

$$\boxed{\vec{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 0 \\ -2 \\ 2 \end{pmatrix} \lambda + \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix} t}$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ 0 & -2 & 2 \\ 1 & -3 & 1 \end{vmatrix} = i\,(-2+6) - j\,(0-2) + k\,(0+2)$$

$$= 4i + 2j + 2k.$$

$$\Rightarrow 4x + 2y + 2z = D. \Rightarrow \boxed{4x + 2y + 2z = 14}$$

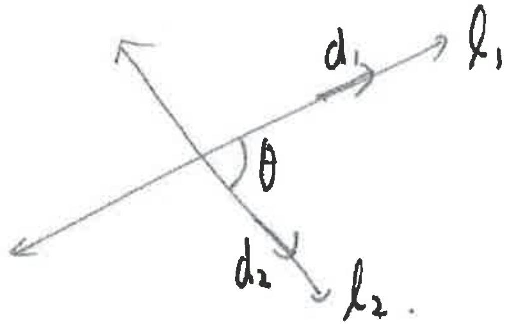
$$D = 4 + 10 = 14$$

b) Area of $\triangle ABC = \frac{1}{2} \left| \vec{AB} \times \vec{AC} \right|$

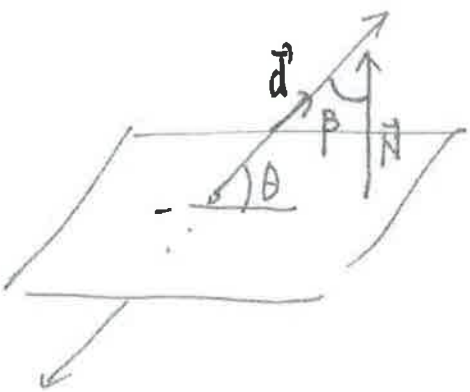
$$= \frac{1}{2}\sqrt{4^2 + 2^2 + 2^2} = \frac{1}{2}\sqrt{24}$$

$$= \sqrt{6}$$

Angles.

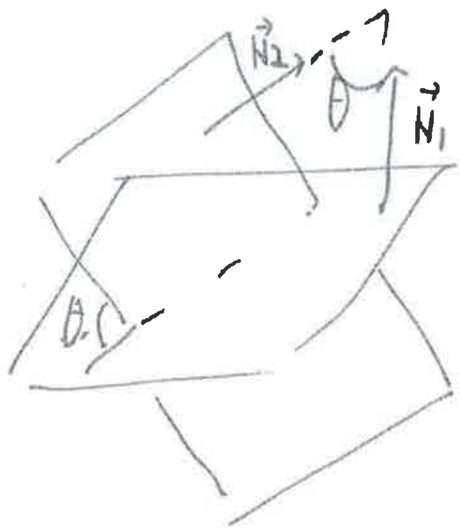


$$\Rightarrow \cos\theta = \left| \frac{d_1 \cdot d_2}{|d_1| \cdot |d_2|} \right|$$



$$\cos\beta = \left| \frac{\vec{d} \cdot \vec{N}}{|\vec{d}|\,|\vec{N}|} \right|$$

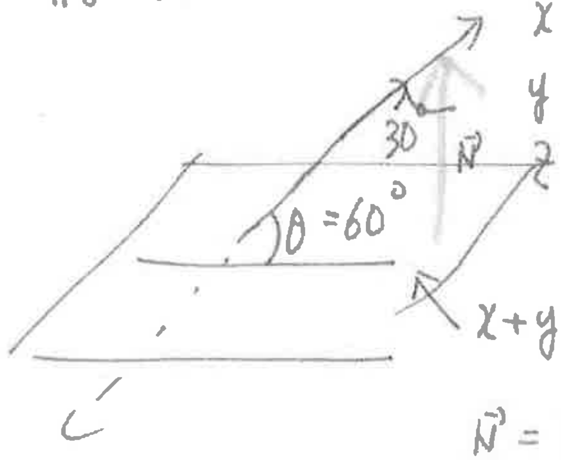
$$\theta = 90^\circ - \beta.$$



$$\cos\theta = \left| \frac{\vec{N_1} \cdot \vec{N_2}}{|\vec{N_1}|\,|\vec{N_2}|} \right|$$

4

#8 d.



$x = 3 + \lambda k$

$y = \lambda$

$z = 1 - \lambda$.

$\vec{d} = \begin{pmatrix} k \\ 1 \\ -1 \end{pmatrix}$

$k = ?$

$x + y = 12$

$\vec{N} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

$\cos 30^\circ = \frac{\sqrt{3}}{2}$

$$\Rightarrow \frac{\vec{N} \cdot \vec{d}}{|\vec{N}|\,|\vec{d}|} = \frac{\sqrt{3}}{2}.$$

$$\Rightarrow \frac{\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} k \\ 1 \\ -1 \end{pmatrix}}{\sqrt{1+1}\,\sqrt{k^2+1+1}} = \frac{\sqrt{3}}{2}.$$

$$\Rightarrow \frac{k+1}{\sqrt{2}\,\sqrt{k^2+2}} = \frac{\sqrt{3}}{2}.$$

$$\Rightarrow (2k+2)^2 = \left(\sqrt{6}\,\sqrt{k^2+2}\right)^2.$$

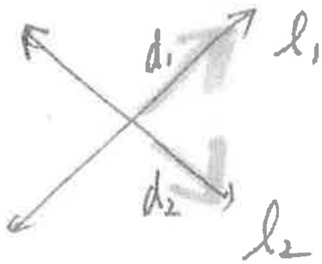
$$4k^2 + 8k + 4 = 6(k^2+2)$$

$$\underline{6k^2 + 12 - 4k^2} - 8k - 4 \Rightarrow 2k^2 - 8k + 8 = 0$$

$$k^2 - 4k + 4 = 0$$

$$(k-2)^2 = 0 \quad k = 2$$

Intersections of lines and planes.



$l_1: x = a_1 + x_1 t$ $\quad$ $l_2: x = a_2 + x_2 \lambda$

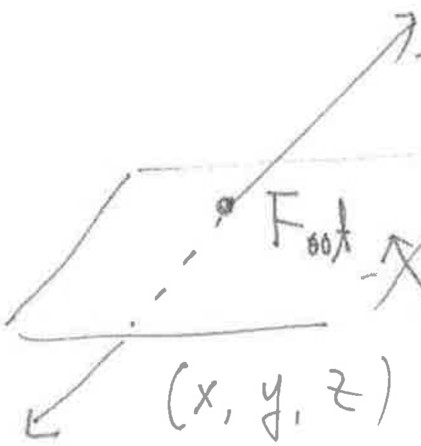
$y = b_1 + y_1 t$ $\quad$ $y = b_2 + y_2 \lambda$

$z = c_1 + z_1 t$ $\quad$ $z = c_2 + z_2 \lambda$

$x = x$ ✓

$y = y$ ✓

$z = z$



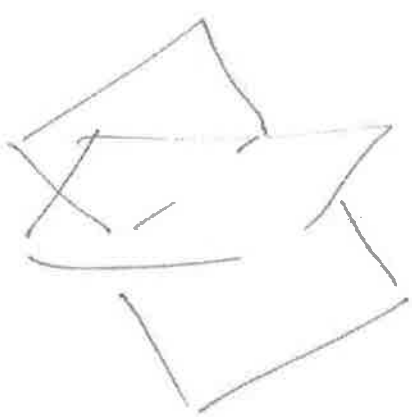
$l_1 \Rightarrow$ $x = a + x_1 t$

$y = b + y_1 t$

$z = c + z_1 t$

$Ax + By + Cz = D$

Find the value of t



Not at this Exam.

Shorest

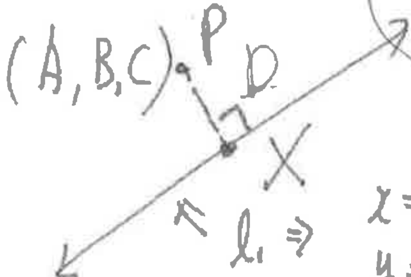
Distances. between ① point to line. ✓

② point to plane

③ line to line ✓ //

④ plane to plane.

$$\frac{\vec{r}\cdot n}{|n|} = \frac{\vec{a}\cdot n}{|n|}$$



$$l_1 \Rightarrow \begin{aligned} x &= a + x_1 t \\ y &= b + y_1 t \\ z &= c + z_1 t \end{aligned} \qquad \Rightarrow \vec{PX} = \begin{pmatrix} a + x_1 t - A \\ b + y_1 t - B \\ c + z_1 t - C \end{pmatrix}$$

$$\vec{d} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$$

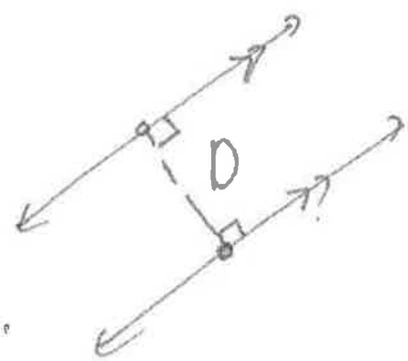
I

$\Rightarrow \vec{PX} \cdot \vec{d} = 0 \Rightarrow$ Find t value.

II. D = distance Formula. ① and ③

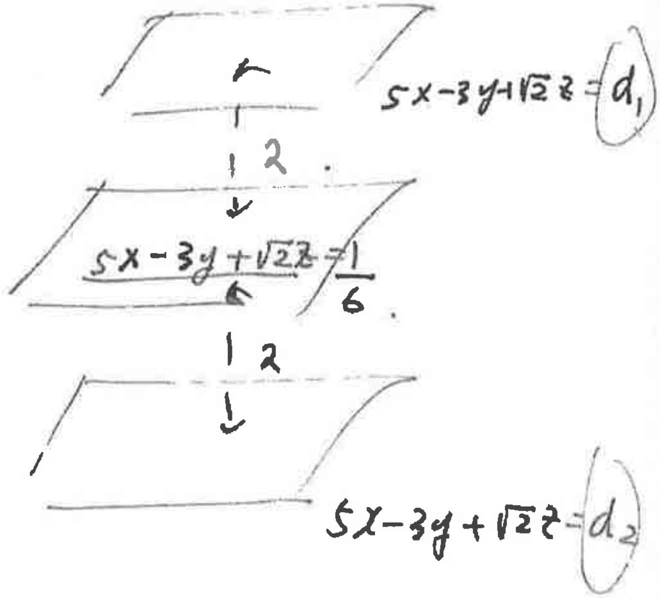
$$D^2 = \sqrt{(a + x_1 t - A)^2 + (b + y_1 t - B)^2 + (c + z_1 t - C)^2}^{\,2}$$

$$D^{2\prime} = 0 \qquad \Rightarrow \text{Solve for } t$$



Same Method as point to line.

Question #3 on Team Quiz.



#3.

$$\frac{\vec{r}\cdot\vec{N}}{|\vec{N}|} = \frac{\vec{a}\cdot\vec{N}}{|\vec{N}|}$$

$$\vec{r}\cdot\hat{n} = \underline{d}$$

$$\vec{r}\begin{pmatrix}5/6\\-3/6\\\sqrt{2}/6\end{pmatrix} = 1/6$$

$$|\vec{N}| = \sqrt{25+9+2} = 6.$$

$$\left|\frac{1}{6} - D\right| = 2. \Rightarrow$$

$$\frac{1}{6} - D = 2 \Rightarrow D = \frac{1}{6} + 2 = \frac{13}{6}$$

$$\frac{1}{6} - D = -2 \Rightarrow D = \frac{1}{6} + 2 = \frac{-11}{6}$$

$$(1)\left(\frac{5x-3y+\sqrt{2}z}{6} = \frac{13}{6}\right)\cdot 6$$

$$(2)\left(\frac{5x-3y+\sqrt{2}z}{6} = \frac{-11}{6}\right)\cdot 6.$$

$$5x-3y+\sqrt{2}z = 13$$

$$5x-3y+\sqrt{2}z = -11.$$