Tuesday (3/21)
Take Notes and then work on practice WS 1) and WS 2)

The answers are on my websites.

Tabular Method

In problems involving repeated applications of integration by parts, a tabular method, illustrated in Example 7, can help to organize the work. This method works well for integrals of the form $\int x^n \sin ax\, dx$, $\int x^n \cos ax\, dx$, and $\int x^n e^{ax}\, dx$.

EXAMPLE 7 Using the Tabular Method

Find $\displaystyle\int x^2 \sin 4x\, dx$.

Solution Begin as usual by letting $u = x^2$ and $dv = v'\,dx = \sin 4x\,dx$. Next, create a table consisting of three columns, as shown.

Alternate Signs	*u and Its Derivatives*	*v′ and Its Antiderivatives*
$+$ →	x^2	$\sin 4x$
$-$ →	$2x$	$-\frac{1}{4}\cos 4x$
$+$ →	2	$-\frac{1}{16}\sin 4x$
$-$ →	0	$\frac{1}{64}\cos 4x$

↑ Differentiate until you obtain 0 as a derivative.

The solution is obtained by adding the signed products of the diagonal entries:

$$\int x^2 \sin 4x\, dx = -\frac{1}{4}x^2\cos 4x + \frac{1}{8}x\sin 4x + \frac{1}{32}\cos 4x + C.$$

Notes: Solving by repeated integrations by parts.

ex) $\int \cos x \cdot e^x\, dx$.

u	dv
$\cos x$	e^x
$-\sin x$	e^x
$-\cos x$	e^x

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$$\boxed{\int \cos x\cdot e^x\,dx} = \cos x\cdot e^x + \sin x e^x - \boxed{\int \cos x e^x\,dx}$$

$$\Rightarrow + \int \cos x\cdot e^x\,dx \qquad\qquad + \int \cos x e^x\,dx$$

$$= \frac{2}{2}\int \cos x\cdot e^x\,dx = \frac{\cos x\cdot e^x + \sin x e^x}{2}$$

$$\Rightarrow \int \cos x\cdot e^x\,dx = \boxed{\frac{1}{2}\left[\cos x e^x + \sin x e^x\right] + C}$$

Practice WS 1)

1. Integrate $\int x\sqrt{9+x}dx$

a. by parts, letting $dv=\sqrt{9+x}dx$.

b. by substitution, letting $u=9+x$.

2. Integrate $\int \frac{x^3}{\sqrt{4+x^2}}dx$

a. by parts, letting $dv=\frac{x}{\sqrt{4+x^2}}dx$.

b. by substitution, letting $u=4+x^2$.

3. Integrate $\int x^2 \sin x dx$ by parts. It will be necessary to repeat the process.

Reminder:

$\frac{d}{dx}(\arcsin x)=\frac{1}{\sqrt{1-x^2}}$ $\frac{d}{dx}(\arccos x)=\frac{-1}{\sqrt{1-x^2}}$ $\frac{d}{dx}(\arctan x)=\frac{1}{1+x^2}$ $\frac{d}{dx}(\operatorname{arcsec} x)=\frac{1}{|x|\sqrt{x^2-1}}$

4. Integrate $\int 4\arccos x dx$ by parts.

5. Evaluate each expression. Give an answer in the indicated interval.

a. $\arcsin\left(\frac{1}{2}\right)$, $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

b. $\arccos\left(-\frac{\sqrt{3}}{2}\right)$, $[0,\pi]$

c. $\arctan(-1)$, $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Practice WS 2)

Find the integral.

1. $\int x^3 \sin x dx$ 2. $\int x^4 e^x dx$ 3. $\int x^2 (x-2)^{\frac{3}{2}} dx$ 4. $\int x^2 e^{x^3} dx$

5. $\int x \sec^2 x dx$ 6. $\int e^{2x} \sin x dx$ 7. $\int e^{-x} \cos 2x dx$ 8. $\int \frac{\cos(\ln x)}{x} dx$

If you want a challenge: evaluate the integral using substitution first, then using integration by parts.

9. $\int \sin\sqrt{x}dx$ [Hint: $w^2=x$, rewrite the integral in terms of w. 10. $\int 2x^3 \cos x^2 dx$

practice WS 1) Solutions

#1. $\int x\sqrt{9-x}\,dx$

a. by parts

$u = x \qquad dv = \sqrt{9-x}\,dx = (9-x)^{\frac{1}{2}}dx$

$du = dx \qquad + \qquad v = -\frac{2}{3}(9-x)^{3/2}$

$-\int$

$$\Rightarrow \frac{-2x}{3}(9-x)^{3/2} + \frac{2}{3}\int (9-x)^{3/2}dx$$

$$= -\frac{2x}{3}(9-x)^{3/2} + \left(\frac{2}{3}\right)\left(-\frac{2}{5}\right)(9-x)^{5/2} + C$$

$$= \boxed{\frac{-2x}{3}(9-x)^{3/2} - \frac{4}{15}(9-x)^{5/2} + C}$$

b. $u = 9-x \quad \Rightarrow \quad x = 9-u.$

$du = -dx$

$$\Rightarrow \int (9-u)\sqrt{u}\,du = \int \left(9u^{1/2} - u^{3/2}\right)du = 9\left(\frac{2}{3}\right)u^{3/2} - \frac{2}{5}u^{5/2} + C$$

$$= \boxed{\frac{18}{3}(9-x)^{3/2} - \frac{2}{5}(9-x)^{5/2} + C}$$

\# 2.

a. by parts.

$u = x^2$ $\quad$ $dv = \dfrac{x}{\sqrt{4+x^2}}\,dx$ $\quad$ $u = 4+x^2$

$du = 2x\,dx$ $\quad$ $du = 2x \cdot dx$

$dv = \int u^{-\frac{1}{2}} \cdot \frac{1}{2}\,du$ $\quad$ $\frac{1}{2}du = x \cdot dx$

$v = \dfrac{2}{2}\sqrt{4+x^2} = \sqrt{4+x^2}$

$$\Rightarrow x^2\sqrt{4+x^2} - \int 2x\sqrt{4+x^2}\,dx.$$

$$= x^2\sqrt{4+x^2} + \frac{2}{3}(4+x^2)^{3/2} + C$$

b. $u = 4+x^2$ $\quad$ $x^2 = u-4$

$du = 2 \cdot x\,dx \Rightarrow \frac{1}{2}du = x\,dx$

$$\int \frac{x^2 \cdot x}{\sqrt{4+x^2}}\,dx = \int \frac{u-4}{\sqrt{u}} \cdot \frac{1}{2}\,du$$

$$= \frac{1}{2}\int (u-4)\,u^{-\frac{1}{2}}\,du = \frac{1}{2}\int \left(u^{\frac{1}{2}} - 4u^{-\frac{1}{2}}\right) du$$

$$= \frac{1}{2} \cdot \frac{3}{2}u^{3/2} - \frac{1}{2} \cdot 8\,u^{\frac{1}{2}} + C$$

$$= \frac{3}{4}(4+x^2)^{3/2} - 4(4+x^2)^{1/2} + C$$

#3. $\int x^2 \sin x \, dx$

u	dv
x^2	$\sin x$
$2x$	$-\cos x$
2	$-\sin x$
0	$+\cos x$

$\oplus \quad \ominus \quad \oplus$

$$\Rightarrow \boxed{-x^2\cos x + 2x\sin x + 2\cos x + C}$$

#4 $\int 4 \arccos x \, dx =$

$u = \arccos x \qquad dv = 4dx$

$du = \dfrac{-1}{\sqrt{1-x^2}}dx \qquad v = 4x$

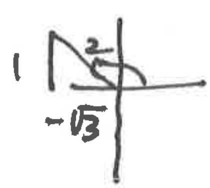
$$\Rightarrow 4x\arccos x + \int \frac{4x}{\sqrt{1-x^2}}\,dx$$

$$\left(\begin{aligned} u &= 1-x^2 \\ du &= -2x\,dx \\ -du &= 2x\,dx \end{aligned} \right)$$

$$= 4x\arccos x - \int \frac{2\,du}{\sqrt{u}}$$

$$= \boxed{4x\arccos x - 4\sqrt{1-x^2} + C}$$

#5. a) $\boxed{\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}}$ b) $\boxed{\arccos\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}}$



c) $\boxed{\arctan(-1) = -\frac{\pi}{4}}$