IB Math HL1: Differential Equations Name: Key Period: ______

1. Separable Equations:

$\boxed{M(x)dx + N(y)dy = 0}$

Example) Solve the differential Equation.

$$xy\frac{dy}{dx} = x^2 + y^2 + x^2y^2 + 1$$

$$xy\frac{dy}{dx} = x^2(y^2+1) + (y^2+1)$$

$$xy\frac{dy}{dx} = (y^2+1)(x^2+1)$$

$$\int \frac{y}{y^2+1}\,dy = \int \frac{x^2+1}{x}\,dx = \int \left(x + \frac{1}{x}\right)dx$$

$u = y^2+1$

$\frac{1}{2}du = y\,dy$

$\times 2$ $\times 2$

$$\frac{1}{2}\ln(y^2+1) = \frac{1}{2}x^2 + \ln x + C$$

$$\ln(y^2+1) = x^2 + 2\ln x + 2C$$

$$y^2+1 = e^{x^2 + \ln x^2 + 2C}$$

$$= e^{x^2} \cdot e^{\ln x^2} \cdot e^{2C}$$

A

$$y^2+1 = A \cdot x^2 \cdot e^{x^2}$$

$$\boxed{y = \pm\sqrt{Ax^2e^{x^2} - 1}}$$

Practice) Find the general solution of the differential equations by separating variables. Write your answer y(x)=

The steps of work is attached.

1. $xydx = (x-5)dy$ $\ln y = x + \ln(x-5)^5 + C$ $\boxed{y = A(x-5)^5 \cdot e^x}$	2. $\frac{dy}{dx} = y\tan x$ $\ln y = \ln\lvert\sec x\rvert + C$ $\boxed{y = A \cdot \sec x}$
3. $(e^{2x}+9)\frac{dy}{dx} = y$ $\frac{dy}{y} = \frac{dx}{e^{2x}+9} \cdot \frac{e^{-2x}}{e^{-2x}}$ $\frac{dy}{y} = \frac{e^{-2x}}{1+9\cdot e^{-2x}}dx$	4. $y\frac{dy}{dx} = e^{x-3y}\cos x \Rightarrow \int ye^{3y}dy = \int e^x\cos x\,dx$ $\frac{1}{3}ye^{3y} - \frac{1}{9}e^{3y} = \frac{1}{2}e^x\cos x + \frac{1}{2}e^x\sin x + C$ $\boxed{e^{3y}(3y-1) = \frac{9}{2}e^x(\cos x + \sin x) + C}$

$\ln y = -\frac{1}{18}\ln\lvert 1+9e^{-2x}\rvert + C \Rightarrow \boxed{y = A(1+9e^{-2x})^{-\frac{1}{18}}}$

MM III: Differential equations by separating variables. Work on graph paper.

Find the general solution of the differential equations by separating variables.

1. $xydx = (x-5)dy$

2. $\dfrac{dy}{dx} = y\tan x$

3. $(e^{2x}+9)\dfrac{dy}{dx} = y$

4. $y\dfrac{dy}{dx} = e^{x-3y}\cos x$

5. $9dx - x\sqrt{x^2-9}\,dy = 0$

6. $xy\dfrac{dy}{dx} = x^2+y^2+x^2y^2+1$

Answers

#1. $\dfrac{x\,dx}{x-5} = \dfrac{dy}{y}$

$\Rightarrow \int\left[1+\dfrac{5}{x-5}\right]dx = \int\dfrac{dy}{y}$

$\Rightarrow \boxed{x+5\ln|x-5| = \ln|y|+c}$

$\boxed{\dfrac{e^x(x-5)^5}{y} = C}$

OR $\boxed{y = A(x-5)^5\cdot e^x.}$

#2 $\int\dfrac{dy}{y} = \int\tan x\,dx$

$\boxed{\ln y = -\ln|\cos x| + c}$ or $\boxed{\ln y = \ln|\sec x| + c}$ $\Rightarrow \boxed{y = A\cdot\sec x}$

#3. $\dfrac{dy}{y} = \dfrac{dx}{e^{2x}+9}$.

$\dfrac{dy}{y} = \dfrac{e^{-2x}dx}{1+9e^{-2x}}$ $\quad u = 1+9e^{-2x}$

$du = -18\,e^{-2x}dx$

$-\dfrac{du}{18} = e^{-2x}dx$.

$\int\dfrac{dy}{y} = \int -\dfrac{1}{18}\,\dfrac{du}{u}$

$\boxed{\ln y = -\frac{1}{18}\ln|1+9e^{-2x}| + c}$ $\Rightarrow \boxed{y = A(1+9e^{-2x})^{\frac{-1}{18}}}$

#4 $y\dfrac{dy}{dx} = \dfrac{e^x}{e^{3y}}\cos x$

$\int y\,e^{3y}\cdot dy = \int e^x\cos x\,dx$

u	dv
y	e^{3y}
1	$\frac{1}{3}e^{3y}$
	$\frac{1}{9}e^{3y}$

u	dv
$\cos x$	e^x
$-\sin x$	e^x
$-\cos x$	e^x

$\frac{1}{3}ye^{3y} - \frac{1}{9}e^{3y}$ $\quad I = e^x\cos x + e^x\sin x - I$

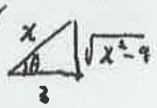
$\Rightarrow \boxed{\frac{1}{3}ye^{3y} - \frac{1}{9}e^{3y} = \frac{1}{2}e^x\cos x + \frac{1}{2}e^x\sin x + c}$

$2I = e^x\cos x + e^x\sin x$

$I = \frac{1}{2}e^x\cos x + \frac{1}{2}e^x\sin x + c$

#5. $9dx = x\sqrt{x^2-9}\,dy$

$\dfrac{9dx}{x\sqrt{x^2-9}} = dy \Rightarrow \dfrac{9(3\sec\theta\tan\theta\,d\theta)}{(3\sec\theta)(3\tan\theta)} = dy$



$3\sec\theta = x \quad 3\tan\theta = \sqrt{x^2-9}$

$dx = 3\sec\theta\tan\theta\,d\theta$

$\int 3\,d\theta = \int dy$.

$3\theta = y + c$

$\boxed{3\sec^{-1}\left(\frac{x}{3}\right) = y + c}$

#6.

$xy\dfrac{dy}{dx} = x^2(1+y^2) + (1+y^2)$

$xy\dfrac{dy}{dx} = (1+y^2)(1+x^2)$

$\int\dfrac{y}{1+y^2}\,dy = \int\dfrac{x^2+1}{x}\,dx$

$u = 1+y^2$ $\quad \boxed{\frac{1}{2}\ln|1+y^2| = \frac{1}{2}x^2 + \ln|x| + c}$

$du = 2y\,dy$