IB Math HL1 The Quotient Rule solutions

Warm Up: Find $\frac{dy}{dx}$

1. $y=\sqrt[8]{3x^7-5x}$

$y' = \frac{1}{8}(3x^7-5x)^{-\frac{7}{8}}(21x^6-5)$

$= \boxed{\frac{(21x^6-5)}{8(3x^7-5x)^{7/8}}}$

2. $y=3x^{15}(2x+8)^{12}$

$y' = 45x^{14}(2x+8)^{12} + 3x^{15}(2)(12)(2x+8)^{11}$

$= \boxed{(45x^{14})(2x+8)^{12} + 72x^{15}(2x+8)^{11}}$

The Quotient Rule

• Given $f(x)=\frac{u(x)}{v(x)} = u \cdot v^{-1}$	**Rewrite $f(x)$ with negative exponent.**
• $\frac{df}{dx} = (u)'v^{-1} + u\cdot(v^{-1})' = u'v^{-1} - u(v^{-2})v'$	**Find $\frac{df}{dx}$ applying the product rule.**
• $\frac{df}{dx} = \frac{u'}{v} - \frac{u\cdot v'}{v^2}$ $= \frac{vu'}{v^2} - \frac{u\cdot v'}{v^2}$	**Simplify by finding the common denominator.**
$\frac{d(\frac{u}{v})}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} = \boxed{\frac{vu'-uv'}{v^2}}$	**The Quotient Rule**

3. Given $y = \dfrac{x^2+1}{2x-5}$ $= (x^2+1)(2x-5)^{-1}$

a) Differentiate y with respect to x using the product rule and simplify the answer.

$$\frac{dy}{dx} = (2x)(2x-5)^{-1} + (x^2+1)[(-1)(2x-5)^{-2}(2)]$$

$$= \frac{(2x-5)2x}{(2x-5)(2x-5)} - \frac{2(x^2+1)}{(2x-5)^2} = \frac{2x(2x-5) - 2(x^2+1)}{(2x-5)^2}$$

$$= \frac{4x^2-10x-2x^2-2}{(2x-5)^2} = \boxed{\frac{2x^2-10x-2}{(2x-5)^2}}$$

b) Differentiate y with respect to x using the Quotient rule and simplify the answer.

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'\cdot v - v'\cdot u}{v^2}$$

$$y = \frac{x^2+1}{2x-5} \Rightarrow \frac{dy}{dx} = \frac{(x^2+1)'(2x-5) - (x^2+1)(2x-5)'}{(2x-5)^2}$$

$$= \frac{2x(2x-5) - (x^2+1)(2)}{(2x-5)^2} = \boxed{\frac{2x^2-10x-2}{(2x-5)^2}}$$

4. Find $\dfrac{dy}{dx}$ for $y = \dfrac{\sqrt{x}}{1-3x}$

$$\frac{dy}{dx} = \frac{(\sqrt{x})'(1-3x) - \sqrt{x}\,(1-3x)'}{(1-3x)^2}$$

$$= \frac{\frac{1}{2}(x)^{-\frac{1}{2}}(1-3x) + 3\sqrt{x}}{(1-3x)^2} = \frac{\left(\frac{(1-3x)}{2\sqrt{x}} + 3\sqrt{x}\right)\cdot 2\sqrt{x}}{\left((1-3x)^2\right)\cdot 2\sqrt{x}}$$

$$= \frac{1-3x+6x}{2\sqrt{x}(1-3x)^2} = \boxed{\frac{1+3x}{2\sqrt{x}(1-3x)^2}} = \boxed{\frac{\sqrt{x}(1+3x)}{2x(1-3x)^2}}$$

Practice)

Find $\frac{df}{dx}$.

1. $f(x)=\frac{x}{\sqrt{2-5x}}$

$$\frac{df}{dx}=\frac{(1)(2-5x)^{\frac{1}{2}}-(x)(-5)\cdot\frac{1}{2}(2-5x)^{-\frac{1}{2}}}{2-5x}$$

$$=\frac{\left(\sqrt{2-5x}+\frac{5x}{2\sqrt{2-5x}}\right)2\sqrt{2-5x}}{(2-5x)\,2\sqrt{2-5x}}$$

$$=\frac{2(2-5x)+5x}{2(2-5x)\sqrt{2-5x}}=\boxed{\frac{4-5x}{2(2-5x)\sqrt{2-5x}}}$$

2. $f(x)=\frac{x^2+5}{\sqrt{3-2x}}$

$$\frac{df}{dx}=\frac{(2x)\sqrt{3-2x}-(x^2+5)\frac{1}{2}(-2)(3-2x)^{-\frac{1}{2}}}{(3-2x)}$$

$$=\frac{\left(2x\sqrt{3-2x}+\frac{(x^2+5)}{\sqrt{3-2x}}\right)\cdot\sqrt{3-2x}}{(3-2x)\cdot\sqrt{3-2x}}$$

$$=\frac{2x(3-2x)+(x^2+5)}{(3-2x)\sqrt{3-2x}}$$

$$=\frac{6x-4x^2+x^2+5}{(3-2x)\sqrt{3-2x}}=\boxed{\frac{-3x^2+6x+5}{(3-2x)\sqrt{3-2x}}}$$

Find the gradient of the tangent line to:

3. $f(x)=x\sqrt{x}+\frac{x}{\sqrt{x^2+1}}$ at $x=1$

4. $f(x)=-2x^3+\frac{x^2-3x}{\sqrt{x+1}}$ at $x=3$

$$f(x)=x^{\frac{3}{2}}+\frac{x}{\sqrt{x^2+1}}$$

$$\frac{df}{dx}=\frac{3}{2}\sqrt{x}+\left[\frac{\sqrt{x^2+1}-x\cdot 2x\cdot\frac{1}{2}(x^2+1)^{-\frac{1}{2}}}{(x^2+1)}\right]\cdot\frac{\sqrt{x^2+1}}{\sqrt{x^2+1}}$$

$$=\frac{3}{2}\sqrt{x}+\frac{x^2+1-x^2}{(x^2+1)\sqrt{x^2+1}}$$

$$\left.\frac{df}{dx}\right|_{x=1}=\frac{3}{2}+\frac{1}{2\sqrt{2}}$$

$$=\frac{3}{2}+\frac{\sqrt{2}}{4}$$

$$=\boxed{\frac{6+\sqrt{2}}{4}}$$

#4 solution attached

(4)

#4 $f(x) = -2x^3 + \frac{x^2-3x}{\sqrt{x+1}}$

$$\frac{df}{dx} = -6x^2 + \frac{\left[(2x-3)(\sqrt{x+1}) - (x^2-3x)(\tfrac{1}{2})(x+1)^{-\frac{1}{2}}\right]2\sqrt{x+1}}{x+1 \quad 2\sqrt{x+1}}$$

$$= -6x^2 + \frac{2(2x-3)(x+1) - (x^2-3x)}{2\sqrt{x+1}(x+1)}$$

$$= -6x^2 + \frac{2(2x^2 + \overset{-x}{2x - \cancel{3x}} - 3) - x^2 + \overset{-2x}{\cancel{3x}}}{2\sqrt{x+1}(x+1)}$$

$$= -6x^2 + \frac{3x^2 + x - 6}{2\sqrt{x+1}(x+1)}$$

27

30
-6
24

$$\left.\frac{df}{dx}\right|_{x=3} = -6(9) + \frac{3(3)^2 + (3) - 6}{2 \cdot 2 \cdot 4}$$

$$= -54 + \frac{24}{\cancel{16}\,4} = -52.5$$