HL Math Review Topic 1: Algebra

Sequences and Series

1. If the sequence $15, 5x-4, \frac{x}{5}$ is arithmetic, what is the exact value of x ?

2. Write each series in sigma notation.

a. $-5-2+1+\ldots+124$

b. $\frac{7}{18}+\frac{14}{19}+\frac{21}{20}+\ldots+\frac{63}{26}$

c. $2+\frac{5}{2}+\frac{25}{8}+\frac{125}{32}+\frac{625}{128}$

3. The first term of an infinite geometric sequence is 98, while the third term is 32. There are two possible sequences. Find the sum of each sequence.

Mathematical Induction

How every induction proof goes:

Step 1: State Proposition ___.

Step 2: Show that the proposition ___ is true.

Step 3: Assume that ___ is true and show that thus ___ is true.

Step 4: Make the conclusion:

"Hence if ___ is true, then ___ is true.

Since ___ is true, then ___ is true for all $a \in \mathbb{Z}$, $n \geq a$."

4. Consider the sum $S_n = \frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}+\ldots+\frac{1}{n\cdot(n+1)}$

a. Make a conjecture about the sum.

b. Use mathematical induction to prove your conjecture.

Complex Numbers

Cartesian Form: $z = a + bi$	**Polar Form:** $z = \|z\| cis\theta$	**Euler's Form:** $z = \|z\| e^{i\theta}$

5. Evaluate a. $4e^{i\left(\frac{-2\pi}{3}\right)}$ b. i^{-i}

DeMoivre's Theorem: $\left(\|z\| cis\theta\right)^n$ = _____ for all rational n.

6. Find the exact value of $\left(\sqrt{3} + i\right)^8$.

The **nth roots of a complex number** z are the solutions of $z^n = c$.

4a. Write 16 in polar form.

b. Hence, find the 4th roots of 16.

- There are exactly ___ n^{th} roots of c.
- If c is real, then the complex roots must occur in _____ pairs.
- The roots of z^n will all have the same modulus which is_____.
- On an Argand diagram, the roots all lie on a circle with radius r = _____, and the roots are equally spaced around that circle.

Linear Systems

7. Describe the possible solutions for the system and when they occur.

a. $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 5 & 6 & 7 \\ 0 & 0 & 14 & 10-2a \end{bmatrix}$ b. $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 5 & 6 & 7 \\ 0 & 0 & 0 & 10-2a \end{bmatrix}$ c. $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 5 & 6 & 7 \\ 0 & 0 & 7-a & 12 \end{bmatrix}$ d. $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 5 & 6 & 7 \\ 0 & 0 & 7-a & 0 \end{bmatrix}$

The Binomial Theorem

8. Find a if the coefficient of x^7 in the expansion of $(3x + a)^{12}$ is -228096.