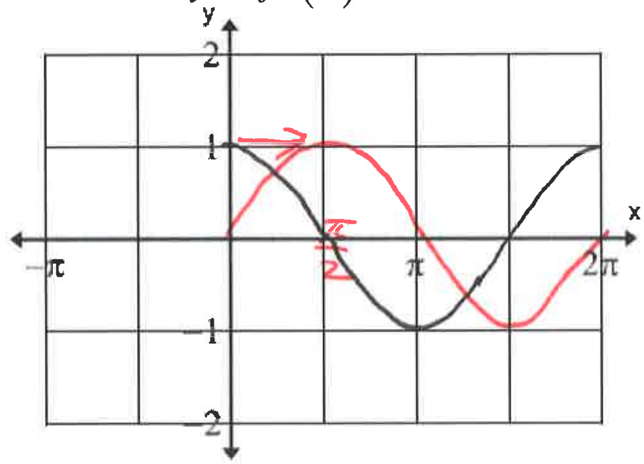
18H Derivatives of Trigonometric Functions

1. Graph $f(x)=\sin x$ on your GFC.

2. Complete the table using the GFC feature. Radian mode

x	Slope of tangent $f(x)=\sin x$
0	1
$\frac{\pi}{2}$	0
π	-1
$\frac{3\pi}{2}$	0
2π	1

3. Use your table to sketch the derivative function $y=f'(x)$.



4. Identify the derivative function:

$f'(x)=$____________

3. Derive $\frac{d}{dx}(\sin x)=\cos x$ by the first principle of Derivative.

$\lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}$	Write $\frac{d}{dx}(\sin x)$ by the first principle.
$=\lim_{h\to 0}\frac{\sin x\cos h+\cos x\sin h-\sin x}{h}$	Apply the compound identity: $\sin(A+B)=\sin A\cos B+\cos A\sin B$
$=\lim_{h\to 0}\frac{\sin x\cos h-\sin x}{h}+\lim_{h\to 0}\frac{\cos x\sin h}{h}$ $=\lim_{h\to 0}\sin x\frac{(\cos h-1)}{h}\frac{(\cos h+1)}{(\cos h+1)}+\cos x\cdot 1$ $=\lim_{h\to 0}\sin x\frac{((\cos h)^2-1)}{h(\cos h+1)}+\cos x$ $=\lim_{h\to 0}\sin x\frac{-\sin^2 h}{h(\cos h+1)}+\cos x$	Simplify and apply trig limit $\lim_{x\to 0}\frac{\sin x}{x}=1$. $\sin x\lim_{h\to 0}\frac{\sin h}{h}\left(\frac{-\sin h}{\cos h+1}\right)+\cos x$ $=(\sin x)(1)\frac{0}{1+1}+\cos x$ $=\boxed{\cos x}$

Trigonometric Derivatives

$\frac{d}{dx}(\sin x) = \cos x$.

$\frac{d}{dx}(\cos x) = \frac{d}{dx}\left(\sin\left(x - \frac{\pi}{2}\right)\right) = \cos\left(x - \frac{\pi}{2}\right)$

$= \boxed{-\sin x}$

$\frac{d}{dx}(\tan x) = \boxed{\sec^2 x}$

$\frac{d}{dx}(\cot x) = \boxed{-\csc^2 x}$

$\frac{d}{dx}(\sec x) =$

$\frac{d}{dx}(\csc x) =$

Examples

$\cancel{\frac{d}{dx}(x\sin x) =}$

$\frac{d}{dx}\tan x = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$

$= \frac{(\cos x)(\cos x) - (-\sin x)\sin x}{(\cos x)^2}$

$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$

$= \frac{1}{\cos^2 x} = \boxed{\sec^2 x}$

$\cancel{\frac{d}{dx}(4\tan^2 3x) =}$

$\cancel{\frac{d}{dx}(\csc(2x^3 + 7x)) =}$

Examples

$\frac{d}{dx}(x\sin x) = \boxed{\sin x + x\cos x}$

$\frac{d}{dx}(4\tan^2 3x) = \boxed{24(\tan 3x)(\sec^2 3x)}$

$\frac{d}{dx} 4[\tan(3x)]^2 = 8(\tan 3x)\sec^2(3x) \cdot 3$

$u \; "u"$

$\frac{d}{dx}(\cos^3(2x)) = 3\cos^2(2x) \cdot (-\sin 2x) \cdot 2$

$= \boxed{-6\cos^2(2x)\sin(2x)}$

$\frac{d}{dx}(\cos(3x^2 + 7))$ u "u"

$= -\sin(3x^2+7)[6x]$

$= \boxed{-6x\sin(3x^2+7)}$.

$\frac{d}{dx}(\sin^5 x) = \frac{d}{dx}(\sin x)^5$

$= 5(\sin x)^4(\cos x)$.