Take Notes ①

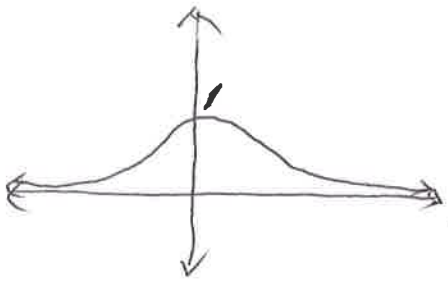
Trigonometric Limits Notes

The Big Idea

$$\lim_{x\to 0}\frac{\sin x}{x} = 1$$

Graphically

$$f(x)=\frac{\sin x}{x}$$



Numerically

x	-0.5	-0.1	-0.01	0	0.01	0.1	0.5
$f(x)=\frac{\sin x}{x}$	.959	.998	.99998	ERROR	.99998	.998	.959

**Because of this result, all of Calculus is done in Radians !!!

* Use the fact $\lim_{u\to 0}\frac{\sin u}{u}=1$

Examples

2. $\lim_{x\to 0}\frac{\sin 3x}{x}$ ← Need to make to be 3x.

$\Rightarrow \lim_{x\to 0}\frac{\sin 3x}{3x}\cdot\frac{3}{1}$

$= \lim_{x\to 0}\frac{\sin 3x}{3x}\cdot\lim_{x\to 0}3 = 1\cdot 3 = \boxed{3}$ (limit = 1)

2. $\lim_{x\to 0}\frac{\sin 4x}{7x}$ ← Need to make to be 4x

$\Rightarrow \lim_{x\to 0}\frac{\sin 4x}{4x}\left(\frac{4x}{7x}\right)$

$= \lim_{x\to 0}\frac{\sin 4x}{4x}\lim_{x\to 0}\frac{4x}{7x} = 1\cdot\frac{4}{7} = \boxed{\frac{4}{7}}$ (limit = 1)

3. $\lim_{x\to 0}\frac{1-\cos x}{x} = \frac{0}{0}$

Multiply with conjugate

$\Rightarrow \lim_{x\to 0}\frac{(1-\cos x)}{x}\frac{(1+\cos x)}{(1+\cos x)}$

$= \lim_{x\to 0}\frac{1-\cos^2 x}{x(1+\cos x)}$ (numerator = $\sin^2 x$) $= \lim_{x\to 0}\frac{\sin^2 x}{x(1+\cos x)}$

$= \lim_{x\to 0}\frac{\sin x}{x}\cdot\lim_{x\to 0}\frac{\sin x}{1+\cos x} = (1)\left(\frac{0}{2}\right) = \boxed{0}$ (limit = 1)

4. $\lim_{x\to 0}\frac{\tan 5x}{\tan 2x} = \lim_{x\to 0}(\tan 5x)(\cot 2x)$

$= \lim_{x\to 0}\left(\frac{\sin 5x}{\cos 5x}\right)\left(\frac{\cos 2x}{\sin 2x}\right)$

$= \lim_{x\to 0}\left(\frac{\sin 5x}{5x}\right)\left(\frac{2x}{\sin 2x}\right)\left(\frac{5x}{2x}\right)\left(\frac{\cos 2x}{\cos 5x}\right)$

$= \lim_{x\to 0}\left(\frac{\sin 5x}{5x}\right)\lim_{x\to 0}\frac{2x}{\sin 2x}\cdot\frac{5}{2}\cdot 1$ (limits = 1 and 1/1)

$= 1\cdot 1\cdot\frac{5}{2}\cdot 1 = \boxed{\frac{5}{2}}$

Answers

1. a. 6 b. 2 c. $\frac{3}{4}$ d. 3 e. $\frac{27}{8}$

2. $\lim_{x\to\frac{\pi}{2}} \sin x = 1$ 3. $\lim_{x\to\pi} \tan x = 0$ 4. $\lim_{x\to 7} \sec\left(\frac{\pi x}{6}\right) = -\frac{2\sqrt{3}}{3}$ 5. $\lim_{x\to 0} \frac{\sin 2x}{x} = 2$

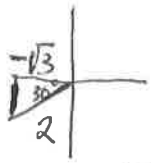
6. $\lim_{x\to 0^+} \frac{\sin x}{\sqrt{x}} = 0$ 7. $\lim_{x\to 0} \frac{\sin 2x}{5x} = \frac{2}{5}$ 8. $\lim_{x\to 0} \frac{1-\cos x}{\sin x} = 0$ 9. $\lim_{\theta\to 0} \frac{\cos\theta \tan\theta}{\theta} = 1$

10. $\lim_{t\to 0} \frac{\tan 5t}{\tan 2t} = \frac{5}{2}$ 11. $\lim_{t\to \pi/2} \frac{\cos x}{\cot x} = 1$ 12. $\lim_{x\to 0} \frac{3(1-\cos x)}{x} = 0$ 13. $\lim_{x\to 0} \frac{\sin^2 x}{x} = 0$

Work:

#2. $\lim_{x\to\frac{\pi}{2}} \sin x = \sin\frac{\pi}{2} = \boxed{1}$

#3 $\lim_{x\to\pi} \tan x = \tan\pi = \boxed{0}$



#4. $\lim_{x\to 7} \sec\left(\frac{\pi\cdot x}{6}\right) = \sec\left(\frac{7\pi}{6}\right) = -\frac{2}{\sqrt{3}} = \boxed{-\frac{2\sqrt{3}}{3}}$

#5. $\lim_{x\to 0} \frac{\sin 2x}{x} = \lim_{x\to 0}\left(\frac{\sin 2x}{2x}\right)\left(\frac{2}{1}\right) = 1\cdot 2 = \boxed{2}$

#6. $\lim_{x\to 0^+} \frac{\sin x}{\sqrt{x}}\cdot\frac{\sqrt{x}}{\sqrt{x}} = \lim_{x\to 0^+} \frac{\sin x}{x}\cdot\sqrt{x} = 1\cdot\sqrt{0} = \boxed{0}$

#7. $\lim_{x\to 0} \frac{\sin 2x}{5x} = \lim_{x\to 0} \frac{\sin 2x}{2x}\left(\frac{2x}{5x}\right) = 1\cdot\frac{2}{5} = \boxed{\frac{2}{5}}$

#8. $\lim_{x\to 0} \frac{(1-\cos x)}{\sin x}\frac{(1+\cos x)}{(1+\cos x)} = \lim_{x\to 0} \frac{1-\cos^2 x}{\sin x(1+\cos x)}$

$= \lim_{x\to 0} \frac{\sin^2 x}{\sin x(1+\cos x)} = \lim_{x\to 0} \frac{\sin x}{1+\cos x} = \frac{0}{1+1} = \boxed{0}$

#9 $\lim_{\theta \to 0} \frac{\cos\theta \cdot \tan\theta}{\theta}$

$= \lim_{\theta \to 0} \frac{\cos\theta}{\theta} \cdot \frac{\sin\theta}{\cos\theta}$

$= \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = \boxed{1}$

#10. $\lim_{t \to 0} \frac{\tan 5t}{\tan 2t} = \lim_{t \to 0} \frac{\sin 5t}{\cos 5t} \cdot \frac{\cos 2t}{\sin 2t}$

$= \lim_{t \to 0} \frac{\sin 5t}{5t} \cdot \frac{2t}{\sin 2t} \left(\frac{5}{2}\right) \frac{\cos 2t}{\cos 5t}$

$= 1 \cdot 1 \cdot \frac{5}{2} \cdot 1 = \boxed{\frac{5}{2}}$

#11. $\lim_{x \to \frac{\pi}{2}} \frac{\cos x}{\cot x} = \lim_{x \to \frac{\pi}{2}} \frac{\cos x}{} \cdot \frac{\sin x}{\cos x}$

$= 1 \cdot 0 = \boxed{0}$

#12. $\lim_{x \to 0} \frac{3(1-\cos x)}{x}$

$= \lim_{x \to 0} \frac{3}{x} \frac{(1-\cos x)(1+\cos x)}{(1+\cos x)}$

$= \lim_{x \to 0} \frac{3}{x} \frac{(1-\cos^2 x)}{1+\cos x}$

$= \lim_{x \to 0} \frac{3}{x} \frac{\sin^2 x}{1+\cos x}$

$= \lim_{x \to 0} 3 \cdot \frac{\sin x}{x} \cdot \frac{\sin x}{1+\cos x} = 3 \cdot 1 \cdot \frac{0}{2} = \boxed{0}$

#13. $\lim_{x \to 0} \frac{\sin x}{x}$

$= \lim_{x \to 0} \frac{\sin x}{x} \cdot \sin x$

$= 1 \cdot 0 = \boxed{1}$