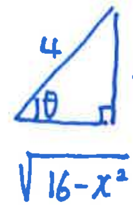
Exit Slip (Trig Substitution) Name: Key Period: ______

1. Using the substitution $x = 4\sin\theta$, Evaluate $\int \frac{\sqrt{16-x^2}}{x}\,dx$

$\boxed{x = 4\sin\theta}$

$\boxed{dx = 4\cos\theta\, d\theta}$



$\boxed{4\cos\theta = \sqrt{16-x^2}}$

$$\int \frac{\sqrt{16-x^2}}{x}\,dx = \int \frac{\cancel{4}\cos\theta}{\cancel{4}\sin\theta}\cdot 4\cos\theta\, d\theta$$

$$= 4\int \frac{\cos^2\theta}{\sin\theta}\,d\theta = 4\int \frac{(1-\sin^2\theta)}{\sin\theta}\,d\theta$$

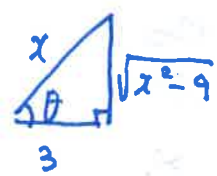
$$= 4\int (\csc\theta - \sin\theta)\,d\theta = -4\ln\left|\csc\theta + \cot\theta\right| - 4\cos\theta + C$$

$$= \boxed{-4\left[\ln\left|\frac{4}{x} + \frac{\sqrt{16-x^2}}{x}\right| + \frac{\sqrt{16-x^2}}{4}\right] + C}$$

2. Using the substitution $x = 3\sec\theta$, Evaluate $\int \frac{\sqrt{x^2-9}}{x}\,dx$

$x = 3\sec\theta$

$dx = 3\sec\theta\tan\theta\, d\theta$



$3\tan\theta = \sqrt{x^2-9}$

$$\int \frac{\sqrt{x^2-9}}{x}\,dx = \int \frac{3\tan\theta}{3\cancel{\sec\theta}}(3\cancel{\sec\theta}\tan\theta)\,d\theta$$

$$= 3\int \tan^2\theta\, d\theta = 3\int (\sec^2\theta - 1)\,d\theta$$

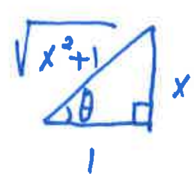
$$= 3[\tan\theta - \theta] + C$$

$$= \boxed{\sqrt{x^2-9} - 3\,\mathrm{arcsec}\left(\frac{x}{3}\right) + C}$$

2. Using the substitution $x = \tan\theta$, Evaluate $\int \frac{9x^3}{\sqrt{1+x^2}}\,dx$

$x = \tan\theta$

$dx = \sec^2\theta\, d\theta$



$\sec\theta = \sqrt{x^2+1}$

$$\int \frac{9x^3}{\sqrt{1+x^2}}\,dx$$

$$= 9\int \frac{\tan^3\theta}{\cancel{\sec\theta}}\sec^{\cancel{2}}\theta\, d\theta = 9\int (\tan\theta)^3\cdot \sec\theta\, d\theta$$

$$\to 9\int \tan^2\theta(\tan\theta\sec\theta)\,d\theta = 9\int (\sec^2\theta - 1)(\tan\theta\sec\theta)\,d\theta$$

$u = \sec\theta \quad du = \sec\theta\tan\theta\, d\theta$

$$\to 9\int (u^2-1)\,du \qquad \rightarrow$$

$$9\int(u^2-1)\,du = 9\left[\tfrac{1}{3}u^3 - u\right] + C \qquad u = \sec\theta$$

$$= 3[\sec\theta]^3 - 9\sec\theta + C$$

$$= \boxed{3\left[x^2+1\right]^{3/2} - 9\sqrt{x^2+1} + C}$$

#1. $\int x^2\sqrt{x+1}\,dx = \int (u-1)^2\sqrt{u}\,du = \int(u^2-2u+1)\sqrt{u}\,du$

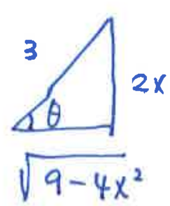
$\left(\begin{array}{l} u = x+1 \rightarrow \quad x = u-1,\ x^2 = (u-1)^2 \\ du = dx \end{array}\right)$

$$= \int\left(u^{\frac{5}{2}} - 2u^{\frac{3}{2}} + u^{\frac{1}{2}}\right)du = \boxed{\tfrac{2}{7}(x+1)^{\frac{7}{2}} - \tfrac{4}{5}(x+1)^{\frac{5}{2}} + \tfrac{2}{5}(x+1)^{\frac{3}{2}} + c}$$

#2. $\int \frac{x+1}{x^2+1}\,dx = \int\frac{x}{x^2+1}\,dx + \int\frac{1}{x^2+1}\,dx$

$$= \boxed{\tfrac{1}{2}\ln|x^2+1| + \arctan x + c}$$

#3 $\int\frac{1}{\sqrt{9-4x^2}}\,dx \Rightarrow \begin{array}{l} 2x = 3\sin\theta \\ dx = \frac{3}{2}\cos\theta\,d\theta \end{array} \Rightarrow$



$3\cos\theta = \sqrt{9-4x^2}$

$$= \int\frac{1}{\cancel{3}\cancel{\cos\theta}}\cdot\frac{\cancel{3}}{2}\cancel{\cos\theta}\,d\theta = \int\tfrac{1}{2}\,d\theta = \tfrac{1}{2}\theta + c = \boxed{\tfrac{1}{2}\arcsin\left(\tfrac{2x}{3}\right) + c}$$