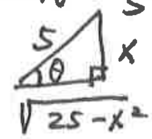
Warm Up

1. $\dfrac{d}{dx}(4\arcsin(4x)) = 16 \cdot \dfrac{1}{\sqrt{1-(4x)^2}} = \boxed{\dfrac{16}{\sqrt{1-16x^2}}}$

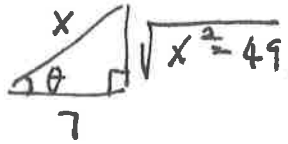
2. $\dfrac{d}{dx}\left(\dfrac{1}{5}\arctan\left(\dfrac{x}{5}\right)\right) = \dfrac{1}{5} \cdot \dfrac{1}{5} \cdot \dfrac{1}{1+\left(\frac{x}{5}\right)^2} = \left(\dfrac{1}{25}\left(\dfrac{1}{1+\frac{x^2}{25}}\right)\right)\dfrac{\cdot 25}{25} = \dfrac{1}{\cancel{25}} \cdot \dfrac{\cancel{25}}{25+x^2} = \boxed{\dfrac{1}{25+x^2}}$

3. Label a right triangle so that $5\sin\theta = x$. $\Rightarrow \sin\theta = \dfrac{x}{5}$.
   Hence, find $\cos\theta$. $\cos\theta = \boxed{\dfrac{\sqrt{25-x^2}}{5}}$



4. Label a right triangle so that $7\sec\theta = x$.
   Hence, find $\tan\theta$.

$\tan\theta = \dfrac{\sqrt{x^2-49}}{7}$ $\quad \sec\theta = \dfrac{x}{7}$

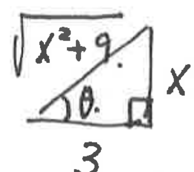


Trigonometric Substitution:
When you can't use u-substitution

if you see this in an integral...	then make this substitution.
$\sqrt{a^2-x^2}$	$x = a\sin\theta$
x^2+a^2	$x = a\tan\theta$
$\sqrt{x^2-a^2}$	$x = a\sec\theta$

$\displaystyle\int \frac{1}{9\left[\left(\frac{x}{3}\right)^2+1\right]}\,d\theta$.

5. $\displaystyle\int \frac{1}{x^2+9}\,dx = \int \frac{1}{x^2+3^2}\,dx$ $\quad x = 3\tan\theta$.



$\tan\theta = \dfrac{x}{3} \Rightarrow x = 3\tan\theta \Rightarrow x^2 = 9\tan^2\theta$.

$dx = 3\sec^2\theta\, d\theta$.

$\tan\theta = \dfrac{x}{3} \Rightarrow x = \arctan\left(\dfrac{x}{3}\right)$

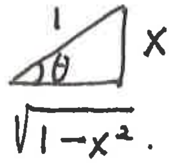
$\displaystyle= \int \frac{1}{9\tan^2\theta+9}(3)(\sec^2\theta\, d\theta)$.

$\displaystyle= \int \frac{1}{\cancel{9}^{\,3}(\tan^2\theta+1)} \cdot \cancel{3}\sec^2\theta\, d\theta$.

$\displaystyle= \int \frac{\cancel{(\tan^2\theta+1)}}{3\cancel{(\tan^2\theta+1)}} \cdot d\theta = \int \frac{1}{3}\,d\theta = \frac{1}{3}\theta + C$

$= \dfrac{1}{3}\arctan\left(\dfrac{x}{3}\right) + C$

6. $\int \frac{x^2}{\sqrt{1-x^2}}\,dx \Leftarrow \begin{cases} x = \sin\theta. \\ dx = \cos\theta\, d\theta \\ \cos\theta = \sqrt{1-x^2} \end{cases}$



$\Rightarrow \theta = \arcsin x.$

$$\int \frac{\sin^2\theta}{\cancel{\cos\theta}} \cancel{\cos\theta}\, d\theta.$$

$$= \int \sin^2\theta\, d\theta.$$

$$= \int \tfrac{1}{2}(1-\cos 2\theta)\, d\theta$$

$$= \tfrac{1}{2}\theta - \tfrac{1}{4}\sin 2\theta + C.$$

$$= \tfrac{1}{2}\theta - \tfrac{1}{2}\sin\theta\cos\theta + C.$$

$$\cos 2\theta = 1 - 2\sin^2\theta.$$

$$-2\sin^2\theta = \cos 2\theta - 1$$

$$2\sin^2\theta = 1 - \cos 2\theta$$

$$\sin^2\theta = \tfrac{1}{2}(1-\cos 2\theta)$$

$$\cos^2\theta = \tfrac{1}{2}(1+\cos 2\theta)$$

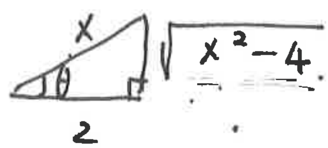
$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\boxed{\tfrac{1}{2}(\arcsin x) - \tfrac{1}{2}\cdot x \cdot \sqrt{1-x^2} + C}$$

7. $\int \frac{\sqrt{x^2-4}}{x}\,dx$

$$x = 2\sec\theta$$

$$\sec\theta = \frac{x}{2}.$$



$$dx = 2\sec\theta\cdot\tan\theta\, d\theta$$

$$\tan\theta = \frac{\sqrt{x^2-4}}{2}.$$

$$2\tan\theta = \sqrt{x^2-4}.$$

$$\int \frac{2\tan\theta}{\cancel{2\sec\theta}}(\cancel{2\sec\theta}\cdot\tan\theta)\, d\theta.$$

$$\Rightarrow \int \sin\theta\,(2\sec\theta\cdot\tan\theta)\, d\theta.$$

$$\int \sin\theta\cdot\left(\frac{2}{\cos\theta}\cdot\frac{\sin\theta}{\cos\theta}\right)$$

$$= \int \frac{\sin^2\theta}{\cos^2\theta}\, d\theta$$

$$= \int 2\tan^2\theta\, d\theta.$$

$$= 2\int(\sec^2\theta - 1)\, d\theta$$

$$= 2\tan\theta - 2\theta + C$$

$$= \boxed{\cancel{2}\left(\frac{\sqrt{x^2-4}}{\cancel{2}}\right) - 2\,\mathrm{arcsec}\left(\frac{x}{2}\right) + C}$$