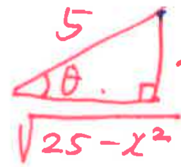
Warm Up

1. $\frac{d}{dx}(4\arcsin 4x)$ $= \frac{4}{\sqrt{1-(4x)^2}} \cdot 4 = \boxed{\frac{16}{\sqrt{1-16x^2}}}$

2. $\frac{d}{dx}\left(\frac{1}{5}\arctan\frac{x}{5}\right)$ $= \frac{1}{5}\left[\frac{1}{1+\left(\frac{x}{5}\right)^2}\right]\cdot\frac{1}{5} = \frac{1}{\cancel{25}}\left[\frac{1}{1+\frac{x^2}{25}}\right]\cdot \cancel{25} = \boxed{\frac{1}{25+x^2}}$

3. Label a right triangle so that $5\sin\theta = x$. $\Rightarrow \sin\theta = \frac{x}{5}$

Hence, find $\cos\theta$.

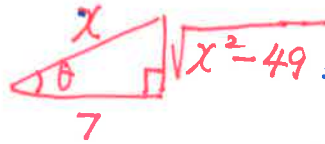


$\Rightarrow \boxed{\cos\theta = \frac{\sqrt{25-x^2}}{5}}$

4. Label a right triangle so that $7\sec\theta = x$. $\Rightarrow \sec\theta = \frac{x}{7}$

Hence, find $\tan\theta$.

$= \cos\theta = \frac{7}{x}$.



$\Rightarrow \boxed{\tan\theta = \frac{\sqrt{x^2-49}}{7}}$

Trigonometric Substitution:

When you can't use u-substitution

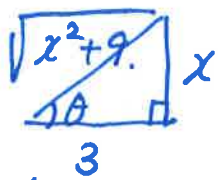
if you see this in an integral...	then make this substitution.
$\sqrt{a^2-x^2}$	$x = a\sin\theta$
x^2+a^2	$x = a\tan\theta$
$\sqrt{x^2-a^2}$	$x = a\sec\theta$

5. $\int \frac{1}{x^2+9}\,dx$

$\tan\theta = \frac{x}{3}$

$x = 3\tan\theta$

$a = \sqrt{9} = 3$



$dx = 3\sec^2\theta\, d\theta$.

$\sec\theta = \frac{\sqrt{x^2+9}}{3} \Rightarrow \left(\sqrt{x^2+9}\right)^2 = (3\sec\theta)^2$

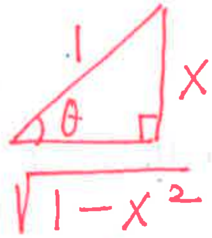
$x^2+9 = 9\sec^2\theta$.

$\int \frac{1}{\cancel{9}_{3}\,\cancel{\sec^2\theta}} \cdot \cancel{3}\,\cancel{\sec^2\theta}\, d\theta$

$\tan\theta = \frac{x}{3} \Rightarrow \theta = \arctan\left(\frac{x}{3}\right)$

$= \int \frac{1}{3}\,d\theta = \frac{1}{3}\theta + C = \boxed{\frac{1}{3}\arctan\left(\frac{x}{3}\right) + C}$

6. $\int \frac{x^2}{\sqrt{1-x^2}}dx$ $\boxed{\sin\theta = \frac{x}{1}}$ $\Rightarrow$



$\Rightarrow$ $\boxed{\cos\theta = \sqrt{1-x^2}}$

$\int \frac{\sin^2\theta}{\cancel{\cos\theta}} \cancel{\cos\theta}\, d\theta$ $\boxed{dx = \cos\theta\, d\theta}$

$= \int \sin^2\theta\, d\theta = \int \frac{1}{2}(1-\cos 2\theta)\, d\theta$

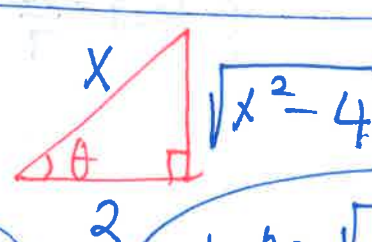
$= \frac{1}{2}\left[\theta - \frac{1}{2}\sin 2\theta\right] + C$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\frac{1}{2}\arcsin x - \frac{1}{2}\cdot x \cdot \sqrt{1-x^2} + c$

7. $\int \frac{\sqrt{x^2-4}}{x}dx$

$\sec\theta = \frac{x}{2}$ $\Rightarrow$



$x = 2\sec\theta$

$\boxed{dx = 2\sec\theta\tan\theta\, d\theta}$

$2\tan\theta = \sqrt{x^2-4}$

$\int \frac{\cancel{2}\tan\theta}{\cancel{2}\cancel{\sec\theta}} \cdot 2\cancel{\sec\theta}\tan\theta\, d\theta$

$= \int 2\tan^2\theta\, d\theta = 2\int(\sec^2\theta - 1)\, d\theta = 2[\tan\theta - \theta] + C$

$= \boxed{\sqrt{x^2-4} - 2\operatorname{arcsec}\left(\frac{x}{2}\right) + C}$

More Substitution

Use the hints to integrate.

1. $\int x^2\sqrt{x+1}\,dx$ [u = x + 1]

2. $\int \frac{x+1}{1+x^2}dx$ [Write as two integrals.]

3. $\int \frac{1}{\sqrt{9-4x^2}}dx$ $[2x = 3\sin\theta]$

Choose an appropriate integration method.

4. $\int \frac{x}{\sqrt{x+4}}dx$

5. $\int_0^{\sqrt{3}/2} \frac{t^2}{(1-t^2)^{3/2}}dt$

6. $\int \sqrt{16-49x^2}\,dx$

7. $\int e^{2x}\sqrt{1+e^{2x}}\,dx$

8. $\int \frac{\ln(x+1)}{x+1}dx$

9. $\int \frac{2x-5}{4x^2}dx$

10. $\int_0^3 \frac{x^3}{\sqrt{x^2+9}}dx$

11. $\int_0^{3/5} \sqrt{9-25x^2}\,dx$

If you want a challenge: evaluate the integral using substitution first, then using trig substitution.

12. $\int e^x\sqrt{1-e^{2x}}\,dx$

13. $\int (x+1)\sqrt{x^2+2x+2}\,dx$