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IB Math HL1 21F and G u-Substitution notes

Warm up:

1. $\int e^{5x-3}\,dx$

$= \boxed{\frac{1}{5}e^{5x-3}+c}$

2. $\int (4-3x)^7\,dx$

$= \frac{-1}{3\cdot 8}(4-3x)^8 + c = \boxed{\frac{-1}{24}(4-3x)^8+c}$

U-Substitution Method: $\Rightarrow$ Rewrite $\int f(x)g(x)dx \Rightarrow \int f(u)\cdot du$ where $du = g(x)dx$

1. $\int (x^2+3x)^4 \boxed{(2x+3)\,dx}$

$u = x^2+3x$

$\frac{du}{dx} = 2x+3$

$\Rightarrow du = (2x+3)dx$

$\int u^4 \cdot du$

$= \frac{1}{5}u^5 + c$

$= \boxed{\frac{1}{5}(x^2+3x)^5+c}$

2. $\int e^{x^2-x}\boxed{(2x-1)\,dx}$

$u = x^2-x$

$du = (2x-1)dx$

$\int e^u \cdot du$

$= e^u + c$

$= \boxed{e^{x^2-x}+c}$

3. $\int \frac{x^2}{x^3-7}dx$

$u = x^3-7$

$du = 3\cdot\boxed{x^2\cdot dx}$

$\frac{1}{3}\cdot du = x^2\cdot du$

$\int \frac{1}{u}\cdot\frac{1}{3}\cdot du = \frac{1}{3}\ln|u| + c$

$= \boxed{\frac{1}{3}\ln|x^3-7|+c}$

4. $\int \frac{(\ln x)^3}{x}dx$

$u = \ln x$

$du = \frac{1}{x}\cdot dx$

$\int u^3\cdot du$

$= \frac{1}{4}u^4 + c$

$= \boxed{\frac{1}{4}(\ln x)^4+c}$

5. Find $f(x)$ where $\frac{df}{dx} = \tan x$ and $f(\frac{\pi}{4}) = 0$

$f(x) = \int \tan x\, dx$

$= \int \frac{\sin x}{\cos x}dx$

$u = \cos x$

$du = -\sin x\, dx$

$-du = \sin x\, dx$

$f(x) = \int \frac{1}{u}\cdot(-du) = -\ln|\cos x| + c = \ln|\sec x| + c$

#6. $\int \sin^2 x\, dx$

#7. $\int \cos^3 x\, dx$

$$f(x) = -\ln|\cos x| + C \Leftarrow \begin{array}{l} x = \frac{\pi}{4} \\ y = 0 \end{array}$$

$$0 = -\ln \cos\left(\frac{\pi}{4}\right) + C$$

$$C = \ln\left(\frac{\sqrt{2}}{2}\right)$$

$$\Rightarrow \boxed{f(x) = -\ln|\cos x| + \ln\left(\frac{\sqrt{2}}{2}\right)}$$

#6. $\int \sin^2 x\, dx = \int (\sin x)^2\, dx$.

$\cos 2x = 1 - 2\boxed{\sin^2 x}$

$-2\sin^2 x = \cos 2x - 1$

$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$$= \int \frac{1}{2}(1 - \cos 2x)\, dx$$

$$= \frac{1}{2}\int (1 - \cos 2x)\, dx = \frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right] + C$$

$$= \frac{1}{2}x - \frac{1}{4}\sin 2x + C.$$ ✓

$$= \frac{1}{2}x - \frac{1}{\cancel{4}_2}\cdot \cancel{2}\sin x \cos x + C$$

$$= \frac{1}{2}x - \frac{1}{2}\sin x \cos x + C$$ ✓

#7 $\int \cos^3 x\, dx = \int \cos^2 x \cdot \cos x\, dx$

$$= \int (1 - \sin^2 x)\boxed{\cos x\, dx}.$$

$u = \sin x$.

$du = \cos x\, dx$

$$\int (1 - u^2)\, du = u - \frac{1}{3}u^3 + C$$

$$= \sin x - \frac{1}{3}\sin^3 x + C$$