key

IB Math HL1 Exit Slip (Various Integration Techniques)

1 $\int x\sqrt{1-5x^2}\,dx$

$u = 1-5x^2$ $\quad \left(du = -10x\,dx \quad -\frac{1}{10}du = x\,dx. \right)$

$= \int -\frac{1}{10}(u)^{\frac{1}{2}}du = \left(-\frac{1}{10}\right)\left(\frac{2}{3}\right)(u)^{3/2} + c$

#1 $= \boxed{-\frac{1}{15}(1-5x^2)^{3/2} + c}$

2 $\int x\left(\sqrt[3]{x+5}\right)dx$

$= \int (u-5)(u)^{\frac{1}{3}}du$ $\quad \left(u = x+5 \quad x = u-5 \quad du = dx \right)$

$= \int \left(u^{\frac{4}{3}} - 5u^{\frac{1}{3}}\right)du$ #2

$= \boxed{\frac{3}{7}(x+5)^{7/3} - 5\cdot\frac{3}{4}(x+5)^{4/3} + c}$

3 $\int \frac{x}{\sqrt{x-1}}dx$

$= \int (u-1)(u)^{-1/2}du$ $\quad \left(u = x-1 \quad x = u+1 \quad du = dx \right)$

$= \int \left(u^{\frac{1}{2}} - u^{-1/2}\right)du = \frac{2}{3}u^{3/2} - 2u^{1/2} + c = \boxed{\frac{2}{3}(x-1)^{3/2} - 2(x-1)^{1/2} + c}$

4 $\int \frac{1}{x}\cos(\ln x)dx$

$\left(u = \ln x \quad du = \frac{1}{x}dx \right)$

$= \int (\cos u)du = \boxed{\sin(\ln x) + c}$

5 $\int \sin^3 2x\,dx$

$u = \cos 2x$

$du = -2\sin 2x\,dx$

$-\frac{1}{2}du = \sin 2x\,dx.$

$= \int \sin^2 2x \sin 2x\,dx$

$= \int (1-\cos^2 2x)\sin 2x\,dx$

$= -\frac{1}{2}\int (1-u^2)du = -\frac{1}{2}\left[u - \frac{1}{3}u^3\right] + c = \boxed{-\frac{1}{2}\cos 2x + \frac{1}{6}\cos^3 2x + c}$

6 $\int xe^{2x}dx$

$u = x \quad dv = e^{2x}dx$

$du = dx \quad v = \frac{1}{2}e^{2x}$

$+$

$-\int$

$= \frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}dx$

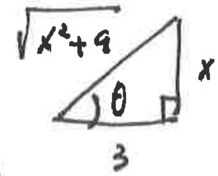
$= \boxed{\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + c}$

7 $\int \frac{\ln x}{x^2}dx$

$$u = \ln x \qquad dv = x^{-2}dx$$
$$du = \frac{1}{x}dx \qquad v = \frac{-1}{x}$$

$$= -\frac{\ln x}{x} + \int \frac{1}{x^2}dx = \boxed{\frac{-\ln x}{x} - \frac{1}{x} + C}$$

8 $\int \frac{x^2}{\sqrt{x^2+9}}dx$



$$\frac{x}{3} = \tan\theta.$$
$$x = 3\tan\theta$$
$$dx = 3\sec^2\theta\, d\theta$$
$$3\sec\theta = \sqrt{x^2+9}.$$
$$\sin\theta = \frac{x}{\sqrt{x^2+9}}.$$

$$= \int \frac{9\tan^2\theta}{\cancel{3}\sec\theta}\cdot \cancel{3}\,\sec^{\cancel{2}}\theta\, d\theta \qquad \sec\theta$$

$$= \int 9\tan^2\theta\sec\theta\, d\theta.$$

$$= 9\int(\sec^2\theta - 1)\sec\theta\, d\theta = 9\int(\sec^3\theta - \sec\theta)d\theta = 9\int \sec^3\theta\, d\theta - 9\int\sec\theta\, d\theta.$$

See attached.

9 $\int\sqrt{9-25x^2}\,dx$

$$= \int\sqrt{(3)^2 - (5x)^2}$$

$$u = 5x$$
$$du = 5dx$$

$$= \frac{1}{5}\int\sqrt{3^2 - u^2}\,du$$

$$\frac{u}{3} = \sin\theta. \quad \sqrt{9-u^2}$$

$$3\cos\theta = \sqrt{9-u^2} \qquad du = +3\cos\theta\, d\theta \Rightarrow$$

$$\frac{1}{5}\int 3\cos\theta\,(+3\cos\theta\, d\theta)$$

$$= +\frac{9}{5}\int \cos\theta^2 d\theta. = +\frac{9}{5}\int\frac{1}{2}(1+\cos 2\theta)d\theta$$

$$= +\frac{9}{10}\left[\theta + \frac{1}{2}\sin 2\theta\right] + C$$

$$= +\frac{9}{10}\left(\theta + \sin\theta\cos\theta\right] + C$$

$$= +\frac{9}{10}\left[\arcsin\left(\frac{5x}{3}\right) + \left(\frac{5x}{3}\right)\frac{\sqrt{9-25x^2}}{3}\right.$$
$$+ C$$

10 $\int e^{2x}\sin(x)dx$

u	dv
e^{2x}	$\sin x$
$2e^{2x}$	$-\cos x$
$4e^{2x}$	$-\sin x$

$$\Rightarrow \int e^{2x}\sin 2x\, dx$$

$$\Rightarrow -e^{2x}\cos x + 2e^{2x}\sin x - 4\int e^{2x}\sin x\, dx$$

$$5\int e^{2x}\sin 2x\, dx = -e^{2x}\cos x + 2e^{2x}\sin x$$

$$\int e^{2x}\sin 2x\, dx = \frac{-1}{5}e^{2x}\cos x + \frac{2}{5}e^{2x}\sin x + C$$

#8 continues.

$$\Rightarrow \int \sec^3\theta \, d\theta$$

$u = \sec\theta \qquad dv = \sec^2\theta \, d\theta$

$\oplus$

$du = \sec\theta\tan\theta \qquad v = \tan\theta .$

$\ominus \int$

$$\int \sec^3\theta\, d\theta = \tan\theta \cdot \sec\theta - \int \sec\theta \tan^2\theta \, d\theta$$

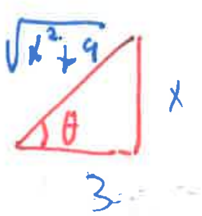
$$= \tan\theta\sec\theta - \int \sec\theta(\sec^2\theta - 1)\, d\theta$$

$$= \tan\theta\sec\theta - \int \sec^3\theta\, d\theta + \int \sec\theta \, d\theta$$

$$2\int \sec^3\theta\, d\theta = \tan\theta\sec\theta + \ln|\sec\theta + \tan\theta| + C$$

$$\int \sec^3\theta\, d\theta = \frac{1}{2}\tan\theta\sec\theta + \frac{1}{2}\ln|\sec\theta + \tan\theta| + C .$$

$$\Rightarrow \frac{9}{2}\left[\tan\theta\sec\theta + \ln|\sec\theta + \tan\theta|\right] + 9\ln|\sec\theta + \tan\theta| + C$$



$$\Rightarrow \frac{9}{2}\left(\frac{x}{3}\right)\left(\frac{\sqrt{x^2+9}}{3}\right) + \frac{27}{2}\ln\left|\frac{\sqrt{x^2+9}}{3} + \frac{x}{3}\right| + C$$

$$= \boxed{\frac{x\sqrt{x^2+9}}{2} + \frac{27}{2}\ln\left|\frac{\sqrt{x^2+9}}{3} + \frac{x}{3}\right| + C}$$