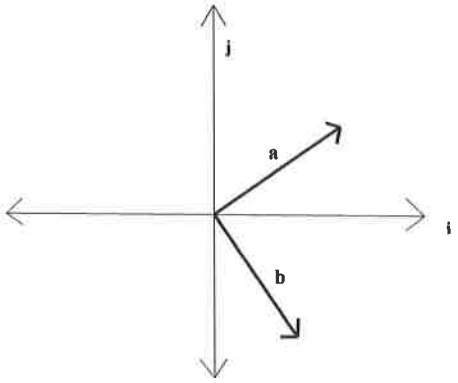
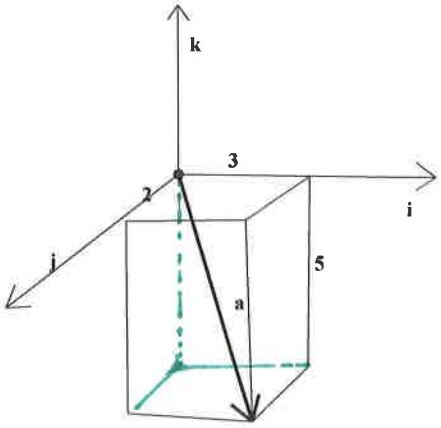
IB Math HL1 : Vector Notes

Two-dimensional (on Plane)	Three-dimensional (on space)
	
• Basis vectors: $\vec{a}=3\vec{i}+2\vec{j}$ and $\vec{b}=2\vec{i}-3\vec{j}$	• Basis vector: $\vec{a}=3\vec{i}+2\vec{j}-5\vec{k}$
• Column Vectors: $\vec{a}=\begin{pmatrix}3\\2\end{pmatrix}$ and $\vec{b}=\begin{pmatrix}2\\-3\end{pmatrix}$	• Column Vectors: $\vec{a}=\begin{pmatrix}3\\2\\-5\end{pmatrix}$
• Magnitude of a vector: $\lvert\vec{a}\rvert=\sqrt{3^2+2^2}=\sqrt{13}$	• Magnitude of a vector: $\lvert\vec{a}\rvert=\sqrt{3^2+2^2+(-5)^2}=\sqrt{38}$
• Unit vector: $\frac{\vec{a}}{\lvert\vec{a}\rvert}=\frac{3i+2j}{\sqrt{3^2+2^2}}=\frac{3i+2j}{\sqrt{13}}$ OR $\frac{3}{\sqrt{13}}i+\frac{2}{\sqrt{13}}j$	• Unit vector: $\frac{\vec{a}}{\lvert\vec{a}\rvert}=\frac{3i+2j-5j}{\sqrt{3^2+2^2+(-5)^2}}=\frac{3i+2j-5j}{\sqrt{38}}$ OR $\frac{3}{\sqrt{38}}i+\frac{2}{\sqrt{38}}j-\frac{5}{\sqrt{38}}k$

Example 1) Find the unit vector of $\vec{v}=5\vec{i}-2\vec{j}+\vec{k}$

unit vector of $\vec{v}=\frac{\vec{v}}{|\vec{v}|}=\frac{5i-2j+k}{\sqrt{(5)^2+(2)^2+(1)^2}}=\boxed{\frac{5}{\sqrt{30}}i-\frac{2}{\sqrt{30}}j+\frac{1}{\sqrt{30}}k}$.

Example 2) Given P (-3, 1, 2) and Q (1, -1, 3)

a) Find $\vec{QP}=\begin{pmatrix}-3\\1\\2\end{pmatrix}-\begin{pmatrix}1\\-1\\3\end{pmatrix}=\boxed{\begin{pmatrix}-4\\2\\-1\end{pmatrix}}$

OR $\boxed{-4i+2j-k}$

b) Find $\left|\frac{1}{2}QP\right|=\frac{1}{2}\begin{pmatrix}-4\\2\\-1\end{pmatrix}=\boxed{\begin{pmatrix}-2\\1\\-\frac{1}{2}\end{pmatrix}}$

OR

$\boxed{-2i+j-\frac{1}{2}k}$

Example 3) Given P(2, 4, -3) and Q(6, -1, 2), find

a. $\overrightarrow{PQ}$ b. The midpoint of $\overline{PQ}$.

$\overrightarrow{PQ} = \begin{pmatrix} 6 \\ -1 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 4 \\ -3 \end{pmatrix} = \begin{pmatrix} 4 \\ -5 \\ 5 \end{pmatrix}$ OR $4\mathbf{i} - 5\mathbf{j} + 5\mathbf{k}$

$= \left(\frac{2+6}{2}, \frac{4-1}{2}, \frac{-3+2}{2} \right)$

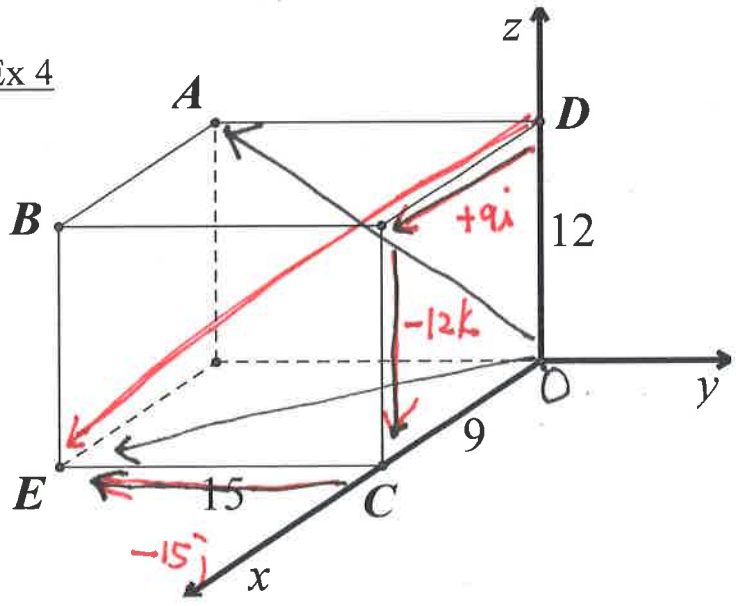
$= \left(4, \frac{3}{2}, \frac{-1}{2} \right)$

Example 4) Given $\mathbf{a} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 5 \\ 2 \\ -3 \end{pmatrix}$, solve for $\mathbf{x}$: $2\mathbf{x} - \mathbf{a} = 3\mathbf{b}$

$2\left(\vec{x}\right) - \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} = 3\begin{pmatrix} 5 \\ 2 \\ -3 \end{pmatrix} \Rightarrow 2\vec{x} = \begin{pmatrix} 15 \\ 6 \\ -9 \end{pmatrix} + \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 18 \\ 5 \\ -7 \end{pmatrix}$

$\vec{x} = \frac{1}{2}\begin{pmatrix} 18 \\ 5 \\ -7 \end{pmatrix} = \begin{pmatrix} 9 \\ \frac{5}{2} \\ \frac{-7}{2} \end{pmatrix}$ OR $9\mathbf{i} + \frac{5}{2}\mathbf{j} - \frac{7}{2}\mathbf{k}$

Ex 4



a. $\overrightarrow{OA} =$ $-15\mathbf{j} + 12\mathbf{k}$

b. $\overrightarrow{OE} =$ $9\mathbf{i} - 15\mathbf{j}$

c. $\overrightarrow{DE} =$ $9\mathbf{i} - 15\mathbf{j} - 12\mathbf{k}$