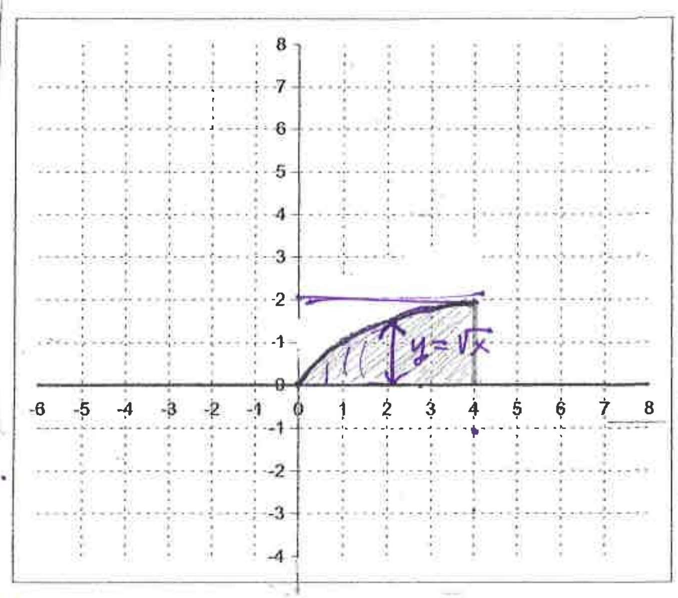
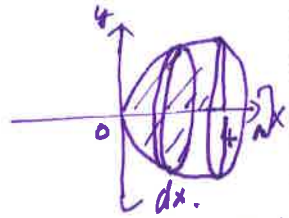
IB Calculus : Volumes of Revolution worksheet Name: ____________________

The graph shows the area bounded by $f(x) = \sqrt{x}$, x-axis , and $x = 4$.



Set up the integrals for the volume that Would be generated by revolving this Curve segments around the following Axes of rotation;

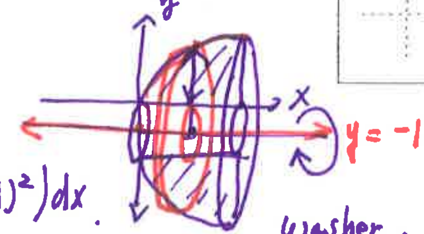
a. x-axis



$$V = \pi \int_0^4 (\sqrt{x})^2 \, dx$$

Disk.

b. y= -1



$$V = \pi \int_0^4 \left((1+\sqrt{x})^2 - (1)^2\right) dx.$$

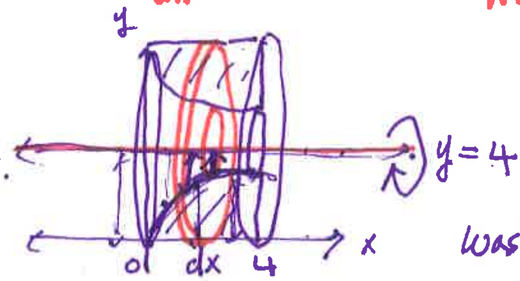
washer.

dx

r_{inner} : 1

R_{outer} : $1+\sqrt{x}$.

c. y= 4



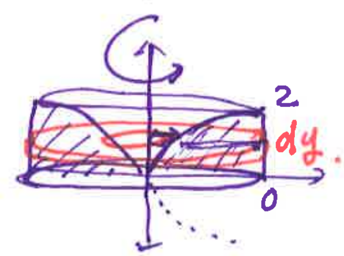
$$V = \pi \int_0^4 \left((4)^2 - (4-\sqrt{x})^2\right) dx.$$

washer

r_{inner} : $4-\sqrt{x}$.

R_{outer} : 4.

d. y-axis



$y=\sqrt{x} \Rightarrow x=y^2$.

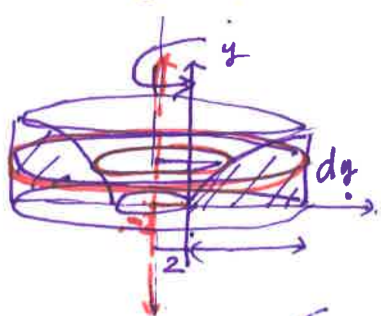
$$V = \pi \int_0^2 \left((4)^2 - (y^2)^2\right) dy.$$

washer

r_{inner} : y^2

R_{outer} : 4

e. x= -2



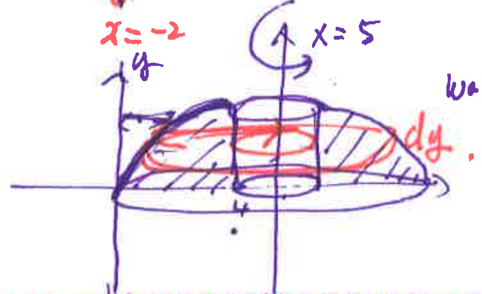
$$V = \pi \int_0^2 \left((6)^2 - (2+y^2)^2\right) dy.$$

washer

r_{inner} : $2+y^2$

R_{outer} : 6.

d. x= ~~4~~ 5



$$V = \pi \int_0^2 \left((5-y^2)^2 - (1)^2\right) dy.$$

washer

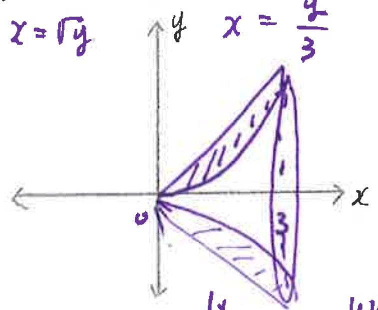
r_{inner} : 1

R_{outer} : $5-y^2$

IB Calculus Exit Slip Name: Key

The region M is enclosed by the function $y = x^2$ and $y = 3x$.

a. Sketch the solid generated by revolving M about the x-axis and set up the integral of the volume.



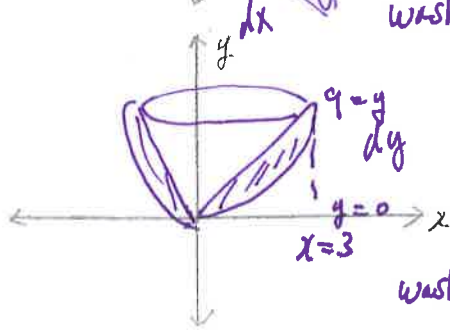
$R_{outer} = 3x$

$r_{inner} = x^2$

$V = \pi \int_0^3 \left((3x)^2 - (x^2)^2\right) dx$

washer

b. Sketch the solid generated by revolving M about the y-axis and set up the integral of the volume.



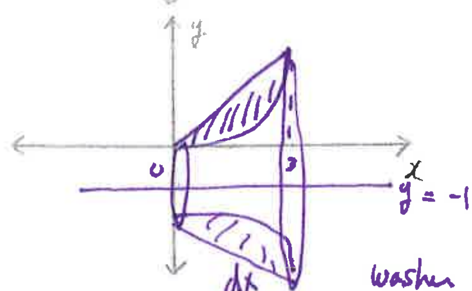
$R_{outer} = \sqrt{y}$

$r_{inner} = \frac{y}{3}$

$V = \pi \int_0^9 \left((\sqrt{y})^2 - \left(\frac{y}{3}\right)^2\right) dy$

washer

c. Sketch the solid generated by revolving M about y= -1 and set up the integral of the volume.



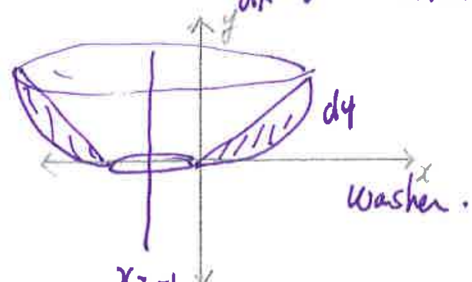
$R_{outer} = 3x+1$

$r_{inner} = x^2+1.$

$V = \pi \int_0^3 \left((3x+1)^2 - (x^2+1)^2\right) dx$

washer

d. Sketch the solid generated by revolving M about x = -1 and set up the integral of the volume.



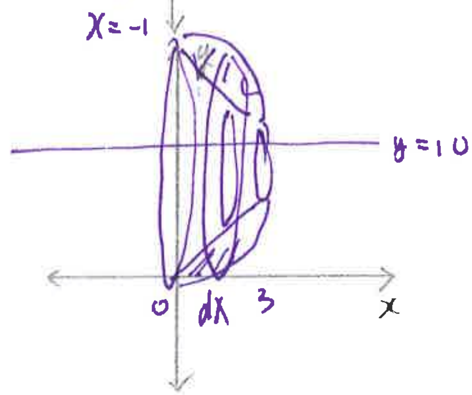
washer.

$R_{outer} = \sqrt{y}+1$

$r_{inner} = \frac{y}{3}+1$

$V = \pi \int_0^9 \left((\sqrt{y}+1)^2 - \left(\frac{y}{3}+1\right)^2\right) dy$

e. Sketch the solid generated by revolving M about y= 10 and set up the integral of the volume.

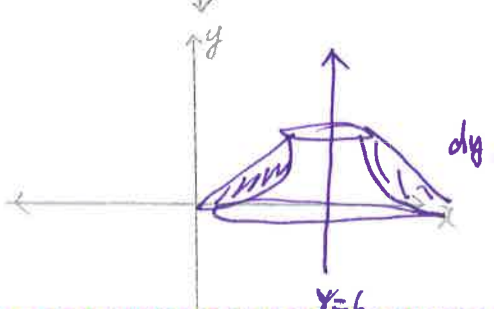


$R_{outer} = 10 - x^2$

$r_{inner} = 10 - 3x$

$V = \pi \int_0^3 \left((10-x^2)^2 - (10-3x)^2\right) dx$

f. Sketch the solid generated by revolving M about x = 6 and set up the integral of the volume.



$R_{outer} = 6 - \frac{y}{3}$

$r_{inner} = 6 - \sqrt{y}$

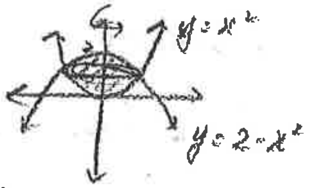
$V = \pi \int_0^9 \left(\left(6-\frac{y}{3}\right)^2 - (6-\sqrt{y})^2\right) dy$

IB Calculus

More practice of Volumes (5 problems)

Name: Key   Period: ______

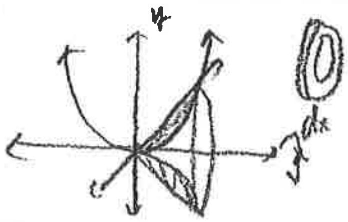
1. The region bounded by the curve $y=2-x^2$ and $y=x^2$ is revolved about y-axis. Sketch the solid and set up the integral of the volume by shell method.



$$\boxed{\cancel{V = 2\pi\int_0^1 x(2-2x^2)\,dx}}$$

$$V = \pi\int_0^1 (\sqrt{y})^2\,dy + \pi\int_1^2 (\sqrt{2-y})^2\,dy$$

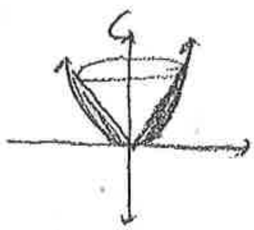
2. The region bounded by the curve $y=x^2$ and $y=4x$ is revolved about x- axis. Sketch the solid and set up the integral of the volume by washer method.



$$\boxed{V=\pi\int_0^4 \left((4x)^2-(x^2)^2\right)dx}$$

3. The region bounded by the curve $y=x^2$ and $y=4x$ is revolved about y- axis. Sketch the solid and set up the integral of the volume by washer method.

$x=\frac{y}{4}$, $x=\sqrt{y}$.

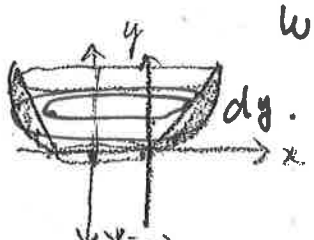


$$\boxed{V=\pi\int_0^{16}\left((\sqrt{y})^2-\left(\frac{y}{4}\right)^2\right)dy}$$

4. The region bounded by the curve $y=x^2$ and $y=4x$ is revolved about x= -2. Sketch the solid and set up the integral of the volume by shell method.

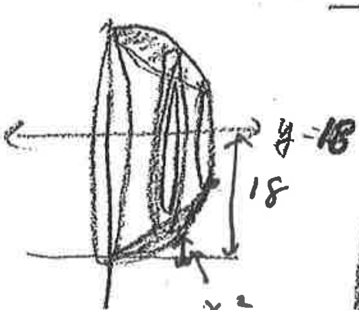
washer.

$$V=\pi\int_0^{16}\left((2+\sqrt{y})^2-\left(2+\frac{y}{4}\right)^2\right)dy$$



$$\boxed{\cancel{V=2\pi\int_0^4 (x+2)(4x-x^2)\,dx}}$$

5. The region bounded by the curve $y=x^2$ and $y=4x$ is revolved about y = ~~4~~ 18. Sketch the solid and set up the integral of the volume by a method of your choice.



Washers

R outer = $(\cancel{4}\,18 - x^2)$

r inner = $(\cancel{4}\,18 - 4x)$

$$\boxed{V=\pi\int_0^4\left((18-x^2)^2-(18-4x)^2\right)dx}$$