Warm up.

Find the complex number z that satisfies the Equation $\frac{50}{z^*} - \frac{10}{z} = 2 + 9i$ given $|z| = 2\sqrt{10}$.

Notes: $|z|^2 = \left(\sqrt{a^2+b^2}\right)^2 = (a+bi)(a-bi) = a^2+b^2 = z \cdot z^*$

$(a+b)(a-b) = a^2 - b^2$

$|z| = 2\sqrt{10}$.

$z^2 = 4 \cdot 10 = 40$

$z \cdot z^* \left(\frac{50}{z^*} - \frac{10}{z} \right) = \left(2 + 9i \right) z \cdot z^*$

$z \cdot 50 - z^* 10 = (2+9i)(40)$

$z = a+bi$

$z^* = a-bi$

$(a+bi)50 - 10(a-bi) = 80 + 360i$

$40a + \boxed{60bi} = 80 + \boxed{360i}$

$40a = 80 \Rightarrow a = 2$

$60b = 360 \Rightarrow b = 6$

IB Math HL2: Complex Numbers Exit Slip

Name: key

No Calculator

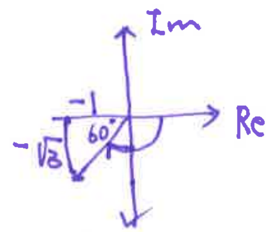
1. Given $z = 4cis(\frac{27\pi}{12})$, write into $a+bi$ where $a,b \in R$

$z = 4\,cis\left(\frac{3\pi}{12}\right) = 4\,cis\left(\frac{\pi}{4}\right) = 4\cos\left(\frac{\pi}{4}\right) + i\,4\sin\left(\frac{\pi}{4}\right)$

$24+3=27.$

$= 4 \cdot \frac{\sqrt{2}}{2} + 4i\frac{\sqrt{2}}{2} = \boxed{2\sqrt{2} + i\,2\sqrt{2}}$

2. Given $z = -1-\sqrt{3}i$, write into a polar form.



$z = 2\,cis\left(-\frac{2\pi}{3}\right)$

$-\pi \leq \theta \leq \pi$

$60°$ OR $\frac{\pi}{3}$

$r = 2$

3. Given $z_1 = 2cis(-\frac{\pi}{3})$ and $z_2 = \sqrt{3}cis(\frac{5\pi}{6})$, find the modulus and argument of $z_1 z_2$.

$z_1 \cdot z_2 = 2\,cis\left(-\frac{\pi}{3}\right)\sqrt{3}\,cis\left(\frac{5\pi}{6}\right)$

$\frac{5\pi}{6} - \frac{2\pi}{6} = \frac{3\pi}{6} = \frac{\pi}{2}$

$= 2\sqrt{3}\,cis\left(\frac{5\pi}{6} - \frac{\pi}{3}\right) = 2\sqrt{3}\left(cis\left(\frac{\pi}{2}\right)\right) \Rightarrow$ Modulus: $2\sqrt{3}$, $Arg(z) = \frac{\pi}{2}$.

4. Find the cubic root of $z = -5i$

$-5i = 5\,cis\left(\frac{3\pi}{2} + 2k\pi\right)$

$\theta = -\frac{\pi}{2}$.

$r = 5$

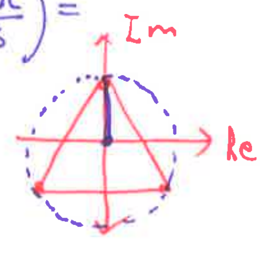
$z^{\frac{1}{3}} = \sqrt[3]{5}\,cis\left[\frac{3\pi}{6} + \frac{2k\pi}{3}\right] = \sqrt[3]{5}\,cis\left(\frac{\pi}{2} + \frac{2k\pi}{3}\right)$

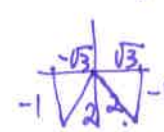
$\frac{4\pi \cdot 2}{3 \cdot 2}$

$k=0$ $\quad z^{\frac{1}{3}} = \sqrt[3]{5}\left[\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right] = \boxed{\sqrt[3]{5}\,i}$

$\frac{3\pi}{6} + \frac{4\pi}{6} = \frac{7\pi}{6}$

$k=1$ $\quad z^{\frac{1}{3}} = \sqrt[3]{5}\left(\cos\left(\frac{7\pi}{6}\right) + i\sin\left(\frac{7\pi}{6}\right)\right) = \boxed{\sqrt[3]{5}\left(\frac{-\sqrt{3}}{2} + \frac{-1}{2}i\right)}$





$k=2$ $\quad z^{\frac{1}{3}} = \boxed{\sqrt[3]{5}\left(\frac{\sqrt{3}}{2} + \frac{-1}{2}i\right)}$

$\frac{3\pi}{6} + \frac{8\pi}{6} = \frac{11\pi}{6}$.