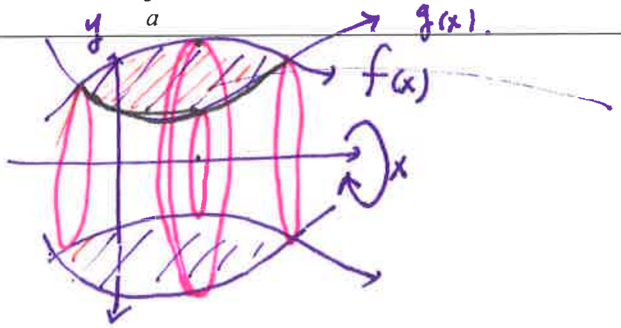
22E.2 The Washer Method

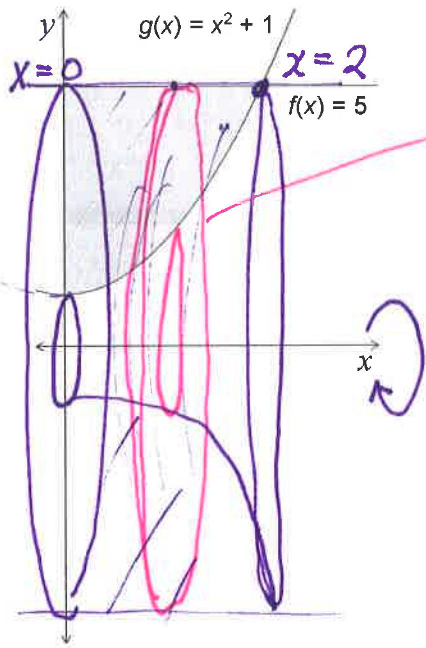
$$Volume = \pi\int_a^b [f^2(x) - g^2(x)]dx$$ where $f(x)$ is outer radius and $g(x)$ is inner radius.



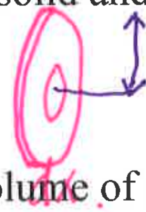
$R_{outer} = f(x)$

$r_{inner} = g(x)$

1. The shaded region is revolved about the x-axis.



a. Sketch the solid and a representative slice of the solid.



$R_{outer} = 5$

$r_{inner} = x^2+1$

b. Find the volume of revolution.

$x^2+1 = 5$

$x^2 - 4 = 0$

$(x-2)(x+2) = 0$

$x = 2 \quad \cancel{x=-2}$

$$V = \pi\int_0^2 \left((5)^2 - (x^2+1)^2\right)dx$$

$$= \pi\int_0^2 \left(25 - x^4 - 2x^2 - 1\right)dx$$

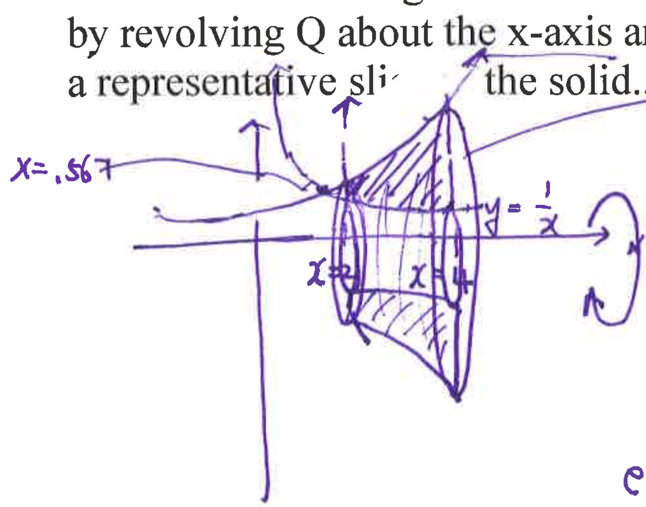
$$= \pi\int_0^2 (-x^4 - 2x^2 + 24)\,dx$$

$$= \pi\left[-\frac{x^5}{5} - \frac{2}{3}x^3 + 24x\right]_{x=0}^{x=2}$$

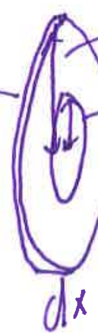
$$= \pi\left[-\frac{2^5}{5} - \frac{2}{3}2^3 + 24\cdot 2\right] = \boxed{\frac{544}{15}\pi}$$

2. The region Q is enclosed by $y = e^x$, $y = \dfrac{1}{x}$, $x = 2$, and $x = 4$.

a. Sketch the solid generated by revolving Q about the x-axis and a representative slice of the solid..



b. Find the volume of revolution.



$R_{outer} = e^x$

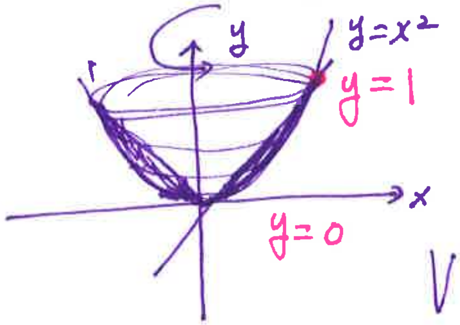
$r_{inner} = \frac{1}{x}$

$$V = \pi\int_2^4 \left((e^x)^2 - \left(\frac{1}{x}\right)^2\right)dx$$

$e^x = \frac{1}{x}$.

$$= \pi\int_2^4 \left(e^{2x} - \frac{1}{x^2}\right)dx.$$

2. Let D be the solid bounded by the parabola $y = x^2$ and the line $y = x$. Find the volume of the solid generated when D is revolved y-axis.



$r_{inner} \Rightarrow y$

$R_{outer} = \sqrt{y}$

$y = y^2$

$y^2 - y = 0$

$y(y-1) = 0$

$y=0 \quad y=1$

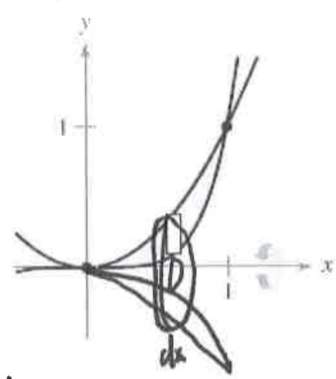
$y = x^2 \Rightarrow x = \pm\sqrt{y}$

$$V = \pi \int_0^1 \left((\sqrt{y})^2 - (y)^2\right) dy.$$

$$= \pi \int_0^1 (y - y^2)\,dy = \pi\left[\frac{y^2}{2} - \frac{y^3}{3}\right]_{y=0}^{y=1} = \pi\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{\pi}{6}$$

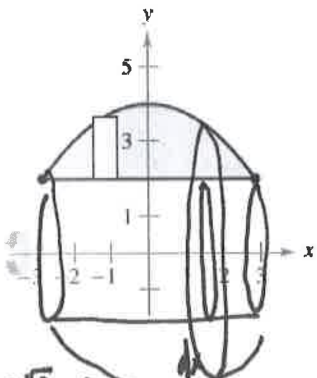
Sketch the solid and set up the integral representing the volume when the region is revolved around the x-axis. DO NOT INTEGRATE!

3. $y = x^2,\ y = x^5$



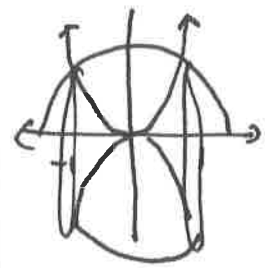
$$\pi \int_0^1 \left((x^2)^2 - (x^5)^2\right) dx$$

4. $y = 2,\ y = 4 - \frac{x^2}{4}$



$$\pi \int_{-\sqrt{8}}^{\sqrt{8}} \left(\left(4 - \frac{x^2}{4}\right)^2 - 2^2\right) dx$$

5. The region bounded by $y = 2 - x^2$ and $y = x^2$



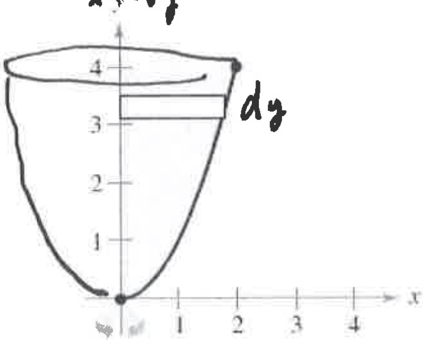
$x = \pm 1$

$$\pi \int_{-1}^{1} \left((2-x^2)^2 - (x^2)^2\right) dx$$

Sketch the solid and set up the integral representing the volume when the region is revolved around the y-axis. DO NOT INTEGRATE!

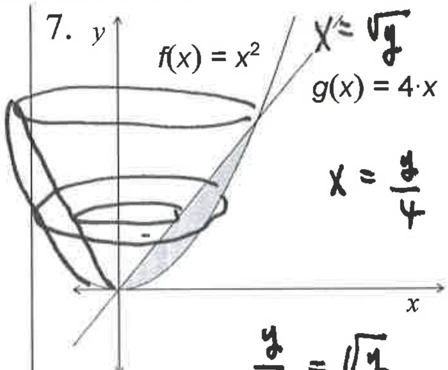
6. $y = x^2$ & $y = 4$

$x = \sqrt{y}$



$$\pi \int_0^4 (\sqrt{y})^2\,dy$$

7.



$x = \sqrt{y}$

$x = \frac{y}{4}$

$\frac{y}{4} = \sqrt{y}$

$y^2 = 16y$

$y^2 - 16y = 0$

$y(y-16) = 0$

$y=0 \quad y=16$

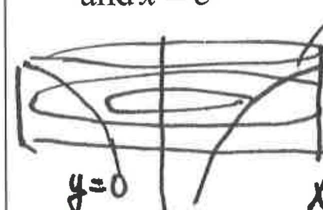
$$\pi \int_0^{16} \left((\sqrt{y})^2 - \left(\frac{y}{4}\right)^2\right) dy$$

8. The region bounded by $y = \frac{1}{3}\ln x$, x-axis, and $x = e^3$

$x = e^{3y}$

$e^3 = e^{3y}$

$y = 1.$



$$\pi \int_0^1 \left((e^3)^2 - (e^{3y})^2\right) dy.$$