

key

MM III: Differential equations by separating variables. Work on graph paper.

Find the general solution of the differential equations by separating variables.

1.  $xydx = (x-5)dy$

3.  $(e^{2x} + 9) \frac{dy}{dx} = y$

5.  $9dx - x\sqrt{x^2 - 9}dy = 0$

2.  $\frac{dy}{dx} = y \tan x$

4.  $y \frac{dy}{dx} = e^{x-3y} \cos x$

6.  $xy \frac{dy}{dx} = x^2 + y^2 + x^2y^2 + 1$

Answers

#1.  $\frac{xdx}{x-5} = \frac{dy}{y}$   
 $\Rightarrow \int [1 + \frac{5}{x-5}] dx = \int \frac{dy}{y}$   
 $\Rightarrow x + 5 \ln|x-5| = \ln|y| + C$   
 $\ln y = x + 5 \ln|x-5| + C$   
 $y = e^{x+5 \ln|x-5|+C}$   
 $y = A \cdot (x-5)^5 \cdot e^x$

#2.  $\int \frac{dy}{y} = \int \tan x dx$   
 $\ln y = -\ln|\cos x| + C$   
 or  $\ln y = \ln|\sec x| + C$   
 $\Rightarrow y = e^{\ln(\sec x) + C} = A \cdot \sec x$

#3.  $\frac{dy}{y} = \frac{dx}{e^{2x} + 9}$

$\frac{dy}{y} = \frac{e^{-2x} dx}{1 + 9e^{-2x}}$

$u = 1 + 9e^{-2x}$   
 $du = -18e^{-2x} dx$   
 $-\frac{dy}{18} = e^{-2x} dx$

$\int \frac{dy}{y} = \int \frac{-1}{18} \frac{du}{u}$

$\ln y = -\frac{1}{18} \ln|1 + 9e^{-2x}| + C$

$\Rightarrow y = A(1 + 9e^{-2x})^{-1/18}$

#5.  $9dx = x\sqrt{x^2 - 9}dy$

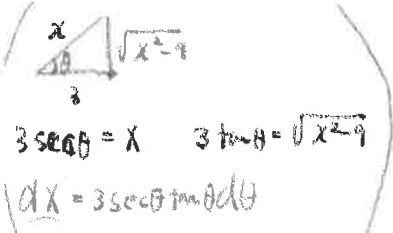
$\frac{9dx}{x\sqrt{x^2 - 9}} = dy$

$\frac{x(3 \sec \theta \tan \theta d\theta)}{(2 \sec \theta)(3 \tan \theta)} = dy$

$3d\theta = dy$

$3\theta = y + C$

$3 \sec^{-1}(\frac{x}{3}) = y + C$



$y = 3 \arcsin(\frac{x}{3}) + C$

#4.  $y \frac{dy}{dx} = \frac{e^x}{e^{2y}} \cos x$

$\int y e^{2y} dy = \int e^x \cos x dx$

u	du
$\frac{1}{3} e^{2y}$	$\frac{2}{3} e^{2y}$
$\frac{1}{9} e^{2y}$	$\frac{2}{9} e^{2y}$
$\frac{1}{27} e^{2y}$	$\frac{2}{27} e^{2y}$

u	du
$\cos x$	$-e^x \sin x$
$- \sin x$	$e^x \cos x$
$- \cos x$	$-e^x \sin x$
$\sin x$	$e^x \cos x$

$\frac{1}{3} y e^{2y} - \frac{1}{9} e^{2y}$

$I = e^x \cos x + e^x \sin x - I$

$2I = e^x \cos x + e^x \sin x$

$I = \frac{1}{2} e^x \cos x + \frac{1}{2} e^x \sin x$

$\Rightarrow \frac{1}{3} y e^{2y} - \frac{1}{9} e^{2y} = \frac{1}{2} e^x \cos x + \frac{1}{2} e^x \sin x + C$

cannot isolate y.

#6.

$xy \frac{dy}{dx} = x^2(1+y^2) + (1+y^2)$

$xy \frac{dy}{dx} = (1+y^2)(1+x^2)$

$\int \frac{y}{1+y^2} dy = \int \frac{x^2+1}{x} dx$

$u = 1+y^2$

$\frac{1}{2} \ln|1+y^2| = \frac{1}{2} x^2 + \ln|x| + C$

$\ln|1+u^2| = x^2 + \ln x^2 + C$

$$\#6. \quad 1+y^2 = e^{(x^2 + \ln x^2 + 2c)} = e^{x^2} \cdot e^{\ln x^2} \cdot e^{2c} \quad (e^{2c} = A)$$

$$= A \cdot x^2 \cdot e^{x^2}$$

$$y^2 = A \cdot x^2 \cdot e^{x^2} - 1$$

$$y = \pm \sqrt{A \cdot x^2 \cdot e^{x^2} - 1}$$