key .

MM III: Differential equations by separating variables. Work on graph paper.

Find the general solution of the differential equations by separating variables.

1. $xydx = (x-5)dy$

2. $\dfrac{dy}{dx} = y\tan x$

3. $(e^{2x}+9)\dfrac{dy}{dx} = y$

4. $y\dfrac{dy}{dx} = e^{x-3y}\cos x$

5. $9dx - x\sqrt{x^2-9}\,dy = 0$

6. $xy\dfrac{dy}{dx} = x^2 + y^2 + x^2y^2 + 1$

Answers

#1. $\dfrac{x\,dx}{x-5} = \dfrac{dy}{y}$

$\Rightarrow \int\left[1+\dfrac{5}{x-5}\right]dx = \int\dfrac{dy}{y}$

$\Rightarrow \boxed{x+5\ln|x-5| = \ln|y| + C}$

$\ln y = x + 5\ln|x-5| + C$

$y = e^{[x+\ln(x-5)^5+c]}$

$\boxed{y = A\cdot(x-5)^5\cdot e^x}$

#2 $\int\dfrac{dy}{y} = \int\tan x\,dx$

$\boxed{\ln y = -\ln|\cos x| + C}$

or $\boxed{\ln y = \ln|\sec x| + C}$

$\Rightarrow y = e^{\ln(\sec x)+C} = \boxed{A\cdot\sec x}$

#3. $\dfrac{dy}{y} = \dfrac{dx}{e^{2x}+9}$

$\dfrac{dy}{y} = \dfrac{e^{-2x}dx}{1+9e^{-2x}}$

$u = 1+9e^{-2x}$

$du = -18e^{-2x}dx$

$-\dfrac{du}{18} = e^{-2x}dx.$

$\int\dfrac{dy}{y} = \int -\dfrac{1}{18}\dfrac{du}{u}$

$\boxed{\ln y = -\frac{1}{18}\ln|1+9e^{-2x}| + C}$

$\Rightarrow \boxed{y = A(1+9e^{-2x})^{-1/18}}$

#4 $y\dfrac{dy}{dx} = \dfrac{e^x}{e^{3y}}\cos x$

$\int ye^{3y}\,dy = \int e^x\cos x\,dx$ (= I)

u	dv
y	e^{3y}
1	$\frac{1}{3}e^{3y}$
0	$\frac{1}{9}e^{3y}$

u	dv
cos x	e^x
− sin x	e^x
− cos x	e^x

$\frac{1}{3}ye^{3y} - \frac{1}{9}e^{3y}$

$I = e^x\cos x + e^x\sin x - I$

$2I = e^x\cos x + e^x\sin x$

$I = \frac{1}{2}e^x\cos x + \frac{1}{2}e^x\sin x$

$\Rightarrow \boxed{\frac{1}{3}ye^{3y} - \frac{1}{9}e^{3y} = \frac{1}{2}e^x\cos x + \frac{1}{2}e^x\sin x + C}$

Cannot Isolate y.

#5. $9dx = x\sqrt{x^2-9}\,dy$

$\dfrac{9dx}{x\sqrt{x^2-9}} = dy \Rightarrow \dfrac{9(3\sec\theta\tan\theta\,d\theta)}{(3\sec\theta)(3\tan\theta)} = dy$

$3\sec\theta = x \qquad 3\tan\theta = \sqrt{x^2-9}$

$dx = 3\sec\theta\tan\theta\,d\theta$

$\int 3\,d\theta = \int dy.$

$3\theta = y + c$

$\boxed{3\sec^{-1}\left(\frac{x}{3}\right) = y + c}$

$\boxed{y = 3\,\mathrm{arcsec}\left(\frac{x}{3}\right) + c}$

#6.

$xy\dfrac{dy}{dx} = x^2(1+y^2) + (1+y^2)$

$xy\dfrac{dy}{dx} = (1+y^2)(1+x^2)$

$\int\dfrac{y}{1+y^2}dy = \int\dfrac{x^2+1}{x}dx$

$u = 1+y^2$

$du = 2y\,du$

$\boxed{\frac{1}{2}\ln|1+y^2| = \frac{1}{2}x^2 + \ln|x| + C}$

$\ln|1+y^2| = x^2 + \ln x^2 + 2C \longrightarrow$

#6. $1+y^2 = e^{(x^2+\ln x^2+2c)} = e^{x^2} \cdot e^{\ln x^2} \cdot e^{2c} \quad (e^{2c}=A)$

$$= A \cdot x^2 \cdot e^{x^2}$$

$$y^2 = A \cdot x^2 \cdot e^{x^2} - 1$$

$$\boxed{y = \pm\sqrt{A \cdot x^2 \cdot e^{x^2} - 1}}$$