IB Math 1 Counting and Factorial Notes

Warm Up

Simplify: $\dfrac{2^{n+3}-2^{n}}{14}$ $= \dfrac{2^n \cdot 2^3 - 2^n}{14}$

$= \dfrac{2^n(2^3-1)}{14}$

$= \dfrac{2^n(7)}{14} = \dfrac{2^n}{2} = 2^{n-1}$

Example 1

Barney has 8 shirts, 5 pairs of pants, and 2 pairs of shoes.

a. How many outfits can Barney create?

$\underset{\text{shirts}}{\underline{8}}$ and $\cdot$ $\underset{\text{pants}}{\underline{5}}$ and $\cdot$ $\underset{\text{shoes}}{\underline{2}}$ $= 80$

AND → multiply
(independent events)

b. How many ways can he choose an item to donate?

$\underset{\text{shirt}}{\underline{8}}$ OR $+$ $\underset{\text{pants}}{\underline{5}}$ OR $+$ $\underset{\text{shoes}}{\underline{2}}$ $= 15$

OR → add
(mutually exclusive)

Example 2

Dara's Deli has a menu consisting of 5 sandwiches, 4 salads, 7 drinks, 3 cakes, and 2 cookies. Customers can get a combo for $6 as follows:

1 sandwich and 1 drink and 1 cake	or	1 salad and 1 drink and 1 cookie	or	1 sandwich and 1 salad and 1 cookie

How many ways can a combo be chosen?

$5 \cdot 7 \cdot 3 + 4 \cdot 7 \cdot 2 + 5 \cdot 4 \cdot 2$

$105 + 56 + 40$

$\boxed{201}$

Example 3

Using the digits 1 – 7, how many 4 digit numbers are there if…

a. repeats allowed

$\underline{7} \cdot \underline{7} \cdot \underline{7} \cdot \underline{7} = 7^4$

$= 2401$

b. no repeats

$\underline{7} \cdot \underline{6} \cdot \underline{5} \cdot \underline{4} = 840$

c. start with an odd # with repeats

$\underline{4} \cdot \underline{7} \cdot \underline{7} \cdot \underline{7} = 1372$

↑ odd #

d. end in an even # with repeats?

$\underline{7} \cdot \underline{7} \cdot \underline{7} \cdot \underline{3} = 1029$

↑ even #

e. end in an even # without repeats?

$\underline{6} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} = 360$

↑ even #

Factorials!

$4!$ $1! = 1$ $0! = 1$

$4 \cdot 3 \cdot 2 \cdot 1$

24

* By definition

Example 4

Simplify: $\dfrac{7! - 6!}{3!} = \dfrac{7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1 - 6\cdot5\cdot4\cdot3\cdot2\cdot1}{3\cdot2\cdot1}$

$= \dfrac{(7-1)\,6\cdot5\cdot4\cdot\cancel{3\cdot2\cdot1}}{\cancel{3\cdot2\cdot1}}$

$= (6)\,6\cdot5\cdot4$

$= 720$

Example 5

Simplify: $\dfrac{(n+1)!-(n-1)!}{n!}$

$$\frac{(n+1)\,n\,\overbrace{(n-1)(n-2)\cdots3\cdot2\cdot1}^{(n-1)!} - \overbrace{(n-1)(n-2)\cdots3\cdot2\cdot1}^{(n-1)!}}{n\,\underbrace{(n-1)(n-2)\cdots3\cdot2\cdot1}_{(n-1)!}}$$

$$\frac{(n+1)n\,(n-1)! - (n-1)!}{n\,(n-1)!}$$

$$\frac{[(n+1)n-1]\,\cancel{(n-1)!}}{n\,\cancel{(n-1)!}} = \boxed{\frac{n^2+n-1}{n}}$$

8A (2-7)

8C.1 (2-6)