IB Math 1 Double and Half Angle Identities

1. Rewrite $\sin 2\theta$ using $\sin(x+y) = \sin x \cos y + \sin y \cos x$.

$\sin(\theta+\theta) = \sin\theta \cdot \cos\theta + \sin\theta \cdot \cos\theta$

$\sin 2\theta = 2(\sin\theta\cos\theta)$

$\sin 2\theta = 2\sin\theta\cos\theta$

2. Rewrite $\cos 2\theta$ using $\cos(x+y) = \cos x \cos y - \sin x \sin y$.

$\cos 2\theta =$

$\cos(\theta+\theta) =$

$\cos\theta\cos\theta - \sin\theta\sin\theta$

$\cos^2\theta - \sin^2\theta$

$\cos^2\theta - (1-\cos^2\theta)$

$2\cos^2\theta - 1$

$(1-\sin^2\theta) - \sin^2\theta$

$1 - 2\sin^2\theta$

3. Rewrite $\tan 2\theta$ using $\tan(x+y)=\dfrac{\tan x+\tan y}{1-\tan x\tan y}$.

$$\tan(\theta+\theta)=\frac{\tan\theta+\tan\theta}{1-\tan\theta\tan\theta}$$

$$=\frac{2\tan\theta}{1-\tan^2\theta}$$

4. Rewrite $\sin\left(\frac{\theta}{2}\right)$ using $\cos 2x = 1 - 2\sin^2 x$.

$$\cos\left(2\cdot\frac{\theta}{2}\right) = 1 - 2\sin^2\left(\frac{\theta}{2}\right)$$

$$\cos\theta = 1 - 2\sin^2\left(\frac{\theta}{2}\right)$$

$$2\sin^2\frac{\theta}{2} + \cos\theta = 1$$

$$2\sin^2\frac{\theta}{2} = 1 - \cos\theta$$

$$\sin^2\frac{\theta}{2} = \frac{1-\cos\theta}{2}$$

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

Thinking required to choose + or −

You can go through the same steps for $\cos\left(\frac{\theta}{2}\right)$ using $\cos 2x = 2\cos^2 x - 1$.

Double Angle Identities

$$\sin 2x = 2\sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\cos 2x = 1 - 2\sin^2 x$$

$$\cos 2x = 2\cos^2 x - 1$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

Half Angle Identities

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$$

$$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$$

$$\tan\frac{x}{2} = \frac{1-\cos x}{\sin x}$$

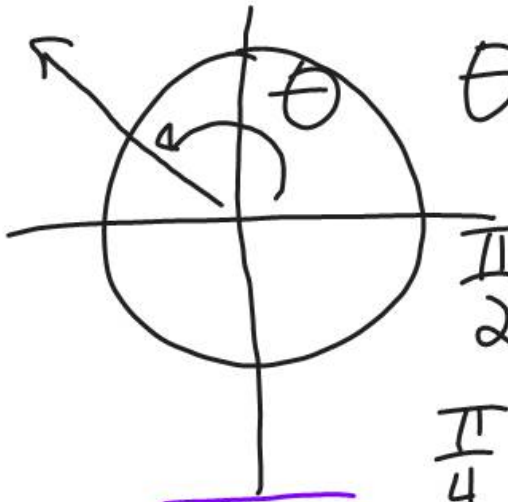
$$\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$$

$$\tan\frac{x}{2} = \frac{\sin x}{1+\cos x}$$

Try This!

5. Given $\sin\theta = \frac{3}{5}$ and $\cos\theta = -\frac{4}{5}$,

find $\sin\left(\frac{\theta}{2}\right)$, $\cos\left(\frac{\theta}{2}\right)$, and $\tan\left(\frac{\theta}{2}\right)$



θ in QII

$\frac{\pi}{2} < \theta < \pi$

$\frac{\pi}{4} < \frac{\theta}{2} < \frac{\pi}{2}$

$\frac{\theta}{2}$ in QI

$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$

QI $= +\sqrt{\frac{1-\left(-\frac{4}{5}\right)}{2}}$

$= \sqrt{\frac{9}{10}}$

$\sin\frac{\theta}{2} = \frac{3\sqrt{10}}{10}$

$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$

QI $= +\sqrt{\frac{1-\frac{4}{5}}{2}}$

$= \sqrt{\frac{1}{10}}$

$\cos\frac{\theta}{2} = \frac{\sqrt{10}}{10}$

$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}$

$= \frac{1-\left(-\frac{4}{5}\right)}{\frac{3}{5}}$

$= \frac{\frac{9}{5}}{\frac{3}{5}} = 3$

$\tan\frac{\theta}{2} = 3$

Trig ID C

save 12, 15 for Tuesday