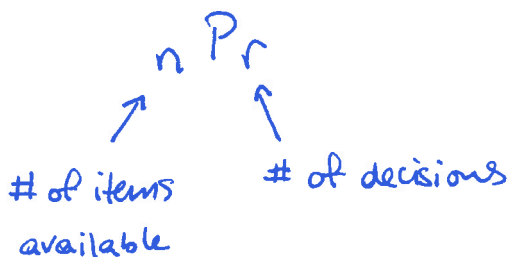


Permutation

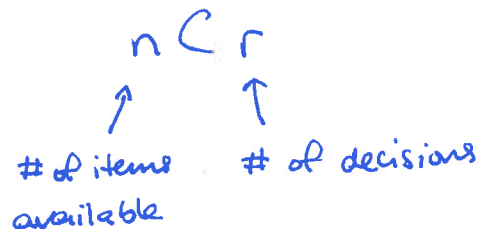
- an arrangement of objects
- order matters



$$nPr = \frac{n!}{(n-r)!}$$

Combination

- a collection of objects
- order does not matter



$$nCr = \frac{n!}{(n-r)!r!} = \frac{nPr}{r!}$$

Example 1

You have 6 books: Math, History, English, French, Physics, and Chemistry. How many ways can they be arranged on a shelf...

a. using all 6 books.

$$\underline{6} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} = 6! = 720$$

Using all 6 books

c. if English and History are together.

$$\begin{array}{c} \boxed{EH} \\ M \\ P \\ F \\ C \end{array} \quad P \quad \boxed{EH} \quad F \quad C \quad M \quad P \quad \boxed{HE} \quad F \quad C \quad M$$

$$(5 \cdot 4 \cdot 3 \cdot 2 \cdot 1) \cdot 2 = \boxed{240}$$

Using all 6 books

d. if Math, English, and History are together.

$$\begin{array}{c} \boxed{MEH} \\ P \\ F \\ C \end{array} \quad P \quad F \quad \boxed{MEH} \quad C \quad \leftarrow 3!$$

$$4! \cdot 3!$$

$$24 \cdot 6 = \boxed{144}$$

b. using 4 of them.

$$\underline{6} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} = \frac{6!}{2!}$$

of books available
of leftover books

$$6P_4$$

Example 2

How many ways can you pick three of your books to carry to the library?

Combination

$$\frac{\# \text{ of permutations of 6 items 3 at a time}}{\# \text{ of perm in a group of 3}} = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = \frac{{}_6P_3}{3!} = \frac{6!}{3! \cdot 3!}$$

MEH HME EHM
MHE HEM EMH

$\boxed{20}$ $= {}_6C_3$

Example 3

The Math Club consists of 5 men and 8 women.

How many ways can a committee be formed with

a. 3 members.

↳ combination

$${}_{13}C_3 = \frac{13 \cdot 12 \cdot 11}{3 \cdot 2 \cdot 1} \leftarrow \# \text{ of permutations}$$

$\leftarrow \# \text{ of ways to arrange 3 things}$

$$= \frac{13!}{10! \cdot 3!}$$

b. 3 women.

$${}_8C_3 = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 56$$

c. 2 men and 4 women.

$${}_5C_2 \cdot {}_8C_4$$
$$\frac{5 \cdot 4}{2 \cdot 1} \cdot \frac{2 \cdot 8 \cdot 7 \cdot 6 \cdot 5}{4 \cdot 3 \cdot 2 \cdot 1}$$
$$10 \cdot 70$$

$\boxed{700}$

Example 4

How many ways can you arrange the letters in...

a. MATH

$$4! = 24$$

b. SASSY

$$\frac{5!}{3!}$$

$$\left. \begin{array}{l} S_1 A S_2 S_3 Y \\ S_2 A S_1 S_3 Y \\ \text{etc.} \end{array} \right\} 3! = 6$$

c. MISSISSIPPI

$$\frac{11!}{4! 4! 2!}$$

d. SASSY using only 4 letters

SASS or SASY or SSSY

$$\frac{4!}{3!} \quad \frac{4!}{2!} \quad \frac{4!}{3!}$$

$$4 + 12 + 4$$

$$\boxed{20}$$

$$8D(1-12)$$
$$8E(1-9)$$