IB Math 1 Permutations and Combinations Notes

Permutation

- an arrangement of objects
- order matters

$_nP_r$ — # of items available; # of decisions

$$_nP_r = \frac{n!}{(n-r)!}$$

Combination

- a collection of objects
- order does not matter

$_nC_r$ — # of items available; # of decisions

$$_nC_r = \frac{n!}{(n-r)!\,r!} = \frac{_nP_r}{r!}$$

Example 1

You have 6 books: Math, History, English, French, Physics, and Chemistry. How many ways can they be arranged on a shelf…

a. using all 6 books.

$6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 6!$

$= 720$

Using all 6 books

b. using 4 of them.

$6 \cdot 5 \cdot 4 \cdot 3 = \frac{6!}{2!}$ (# of books available; $(6-4)!$ # of leftover books)

$_6P_4$

c. if English and History are together.

[EH] M P F C

P [EH] F C M — P [HE] F C M

$(5 \cdot 4 \cdot 3 \cdot 2 \cdot 1) \cdot 2 = \boxed{240}$

Using all 6 books

d. if Math, English, and History are together.

[MEH] P F C

P F [MEH] C (3!)

$4!\ 3!$

$24 \cdot 6 = \boxed{144}$

Example 2

How many ways can you pick three of your books to carry to the library?

Combination

$$\frac{\#\text{ of permutations of 6 items 3 at a time}}{\#\text{ of perm in a group of 3}} = \frac{6\cdot5\cdot4}{3\cdot2\cdot1} = \frac{{}_6P_3}{3!} = \frac{6!}{3!\,3!} = {}_6C_3$$

MEH HME EHM
MHE HEM EMH

20

Example 3

The Math Club consists of 5 men and 8 women.
How many ways can a committee be formed with

a. 3 members. ↳ combination

$${}_{13}C_3 = \frac{13\cdot12\cdot11}{3\cdot2\cdot1} = \frac{13!}{10!\,3!}$$

(13·12·11: # of permutations; 3·2·1: # of ways to arrange 3 things)

b. 3 women.

$${}_8C_3 = \frac{8\cdot7\cdot6}{3\cdot2\cdot1} = 56$$

c. 2 men and 4 women.

$${}_5C_2 \cdot {}_8C_4$$

$$\frac{5\cdot4}{2\cdot1} \cdot \frac{8\cdot7\cdot6\cdot5}{4\cdot3\cdot2\cdot1}$$

$$10 \cdot 70$$

700

Example 4

How many ways can you arrange the letters in…

a. MATH

$4! = 24$

b. SASSY

$\frac{5!}{3!}$

$S_1 A S_2 S_3 Y$

$S_2 A S_1 S_3 Y$

etc.

$3! = 6$

c. MISSISSIPPI

$\frac{11!}{4!\,4!\,2!}$

d. SASSY using only 4 letters

SASS or SASY or SSSY

$\frac{4!}{3!}$ $\frac{4!}{2!}$ $\frac{4!}{3!}$

$4 + 12 + 4$

20

8D (1-12)

8E (1-9)