Warm-up

1) Find the coefficient of x^3 in the expansion of $\left(2x - \frac{1}{x^3}\right)^{11}$.

$${}_{11}C_0\,(2x)^{11} + {}_{11}C_1\,(2x)^{10}\left(\frac{-1}{x^3}\right) + {}_{11}C_2\,(2x)^{9}\left(\frac{-1}{x^3}\right)^{2}$$

$${}_{11}C_2\,(512x^9)\left(\frac{1}{x^6}\right)$$

$$55\,(512\,x^3)$$

$$28160\,x^3$$

$$\boxed{28200\,x^3}$$

$$\boxed{28,200}$$

Section 8G : Binomial Theorem

$(x+y)^n \rightarrow$ Find the m term

$$\left({}_nC_{m-1}\right) x^{n-m+1} y^{m-1}$$

example 1: The coefficient of x^4 in the expansion $(x+a)^6$ is 60. Find a.

$${}_6C_0 x^6 + {}_6C_1 x^5 a + {}_6C_2 x^4 a^2$$

$${}_6C_2 x^4 a^2 = 60x^4$$

$${}_6C_2 a^2 = 60$$

$$15a^2 = 60$$

$$a^2 = 4$$

$$\boxed{a = \pm 2}$$

example 2: The coefficient of x^4 in the expansion $(1-2x)^n$ is 560. Find n.

$${}_nC_0(1)^n + {}_nC_1(1)^{n-1}(-2x) + {}_nC_2(1)^{n-2}(-2x)^2 + {}_nC_3(1)^{n-3}(-2x)^3$$

$$+ {}_nC_4 1^{n-4}(-2x)^4$$

$${}_nC_4(1)^{n-4}(-2x)^4 = 560x^4$$

$${}_nC_4(1)(-2x)^4 = 560x^4$$

$$\frac{n!}{(n-4)!4!}(16x^4) = 560x^4$$

$$\frac{n!}{(n-4)!24} = 35$$

$$\frac{n!}{(n-4)!} = 840$$

$$\frac{n(n-1)(n-2)(n-3)(n-4)!}{(n-4)!} = 840$$

$$n(n-1)(n-2)(n-3) = 840$$

$$n(n-1)(n-2)(n-3) - 840 = 0$$

* plug into graphing calc to find zeros *

zeros are $\boxed{n=7}$ $\cancel{n=-4}$

example 3: Find the term in x^5 in the expansion of
$(3x+A)(2x+B)^6$

$$(3x+A)\left[{}_6C_0(2x)^6 + {}_6C_1(2x)^5B + {}_6C_2(2x)^4B^2 + {}_6C_3(2x)^3B^3 \ldots\right]$$

$$3x\left({}_6C_2(2x)^4B^2\right) + A\left({}_6C_1(2x)^5B\right)$$

$$3x\left(15(16x^4)B^2\right) + A\left(6(32x^5)B\right)$$

$$720x^5B^2 + 192x^5AB$$

$$\boxed{(720B^2 + 192AB)x^5}$$

example 4: Find the constant term in the expansion of
$\left(x-\frac{2}{x}\right)^4\left(x^2-\frac{2}{x}\right)^3$

$$\left({}_4C_0x^4 + {}_4C_1x^3\left(\frac{-2}{x}\right) + {}_4C_2x^2\left(\frac{-2}{x}\right)^2 + {}_4C_3x\left(\frac{-2}{x}\right)^3 \ldots\right)$$

$$\left({}_3C_0x^6 + {}_3C_1x^4\left(\frac{-2}{x}\right) + {}_3C_2x^2\left(\frac{-2}{x}\right)^2 + {}_3C_3\left(\frac{-2}{x}\right)^3\right)$$

$$\text{Constant} = \left({}_4C_2x^2\left(\frac{-2}{x}\right)^2\right)\left({}_3C_2x^2\left(\frac{-2}{x}\right)^2\right)$$

$$\left(6x^2\left(\frac{4}{x^2}\right)\right)\left(3x^2\left(\frac{4}{x^2}\right)\right)$$

$$(24)(12)$$

$$\boxed{288}$$

ex 5: Mina + Norbert each have a regular 6 sided die.

Mina throws her die once and eats a cookie if she throws a 4, 5 or 6.

Norbert throws his die 6 times and eats a cookie if he throws a 5 or 6.

Calculate the probability 5 cookies are eaten.

Two ways:

(1) Mina eats 1, Norbert eats 4

$$\left(\frac{1}{2}\right)\left({}_6C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^2\right)$$

$$\left(\frac{1}{2}\right)\left(15\left(\frac{1}{81}\right)\left(\frac{4}{9}\right)\right)$$

$$\boxed{\frac{60}{1458}}$$

(2) Norbert eats 5

$$\left(\frac{1}{2}\right)\left({}_6C_5\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^1\right)$$

$$\frac{1}{2}\left(6\left(\frac{1}{243}\right)\left(\frac{2}{3}\right)\right)$$

$$\boxed{\frac{12}{1458}}$$

$$\frac{60}{1458} + \frac{12}{1458} = \boxed{\frac{72}{1458}} = \boxed{\frac{4}{81}}$$