

Section 8c.2/8F: Binomial Expansion

Pascal's Δ :

row 0 \rightarrow 1 \swarrow col. 0
row 1 \rightarrow 1 1 \swarrow col. 1

		1										
			1	2	1							
			1	3	3	1						
			1	4	6	4	1					
		1	5	10	10	5	1					
	1	6	15	20	15	6	1					
	1	7	21	35	35	21	7	1				

row 7, col 2

$${}^7C_2 = \frac{7!}{5!2!} = \frac{7 \cdot 6}{2} = 21$$

ex 1: expand $(x+y)^7$

$$\begin{aligned} & {}^7C_0 x^7 y^0 + {}^7C_1 x^6 y^1 + {}^7C_2 x^5 y^2 + {}^7C_3 x^4 y^3 \\ & + {}^7C_4 x^3 y^4 + {}^7C_5 x^2 y^5 + {}^7C_6 x y^6 + {}^7C_7 x^0 y^7 \end{aligned}$$

$$\begin{aligned} = & 1x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 \\ & + 21x^2y^5 + 7xy^6 + 1y^7 \end{aligned}$$

ex 2: What is the 9th term of $(x+y)^n$.

$${}^n C_0 x^n y^0 + {}^n C_1 x^{n-1} y^1 + {}^n C_2 x^{n-2} y^2 + {}^n C_3 x^{n-3} y^3 + \dots + {}^n C_{n-1} x^1 y^{n-1} + {}^n C_n x^0 y^n$$

$$\boxed{{}^n C_8 x^{n-8} y^8}$$

ex 3: Find the 8th term in the expansion of

$$\left(3x - \frac{2}{x^2}\right)^{15}$$

$${}^{15} C_7 (3x)^{15-7} \left(-\frac{2}{x^2}\right)^7$$

$$\frac{15!}{8!7!} (3x)^8 \left(-\frac{2}{x^2}\right)^7$$

$$(6435) (6561 x^8) \left(-\frac{128}{x^{14}}\right)$$

$$- 5,404,164,480 \frac{x^8}{x^{14}}$$

$$\boxed{- 5,404,164,480 x^{-6}}$$

ex 4: What is the coefficient
of x^6 in $(x^2 + \frac{4}{x})^{12}$

$${}_{12}C_6 (x^2)^6 \left(\frac{4}{x}\right)^6$$

$$\frac{12!}{6!6!} (x^{12}) \left(\frac{4096}{x^6}\right)$$

$$924 (x^{12}) \left(\frac{4096}{x^6}\right)$$

$$3,784,704 x^6$$

$$\boxed{3,784,704}$$