Section 7F : Arithmetic Series

Sum of arithmetic series

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Derive the formula

$$S_n = a_1 + (a_1 + d) + (a_1 + 2d) \ldots + (a_1 + (n-1)d)$$

$$+ \quad S_n = a_n + (a_n - d) + (a_n - 2d) \ldots + (a_n - (n-1)d)$$

$$2S_n = (a_1 + a_n)n$$

$$S_n = \frac{(a_1 + a_n)n}{2}$$

ex 1: Find the sum of the first 10 terms of the arithmetic sequence 116, 106, 96 ....

$d = -10$

$$u_{10} = 116 - (10-1)(10)$$

$$u_{10} = 26$$

$$S_{10} = \frac{10}{2}(116 + 26)$$

$$\boxed{S_{10} = 710}$$

ex 2: Find the sum

a) $\sum_{n=1}^{20} 2n-1$

$u_1 = 1$

$u_{20} = 39$

$S_{20} = \frac{20}{2}(1+39)$

$\boxed{S_{20} = 400}$

b) Find n if $\sum_{n=1}^{n} 3n-11 = 5536$

$S_n = \frac{n}{2}(a_1 + a_n)$

$5536 = \frac{n}{2}(-8 + (3n-11))$

$5536 = \frac{n}{2}(-19+3n)$

$11072 = n(-19+3n)$

$11072 = -19n + 3n^2$

$0 = 3n^2 - 19n - 11072$

$\frac{19 \pm \sqrt{19^2 - 4(3)(-11072)}}{6}$

$\frac{19 \pm 365}{6}$

$\boxed{n = 64}$ $n = -57.67$