Section 7G : Geometric Series

Sum of a Geometric Series

$$S_n = \frac{u_1(1-r^n)}{1-r}$$

* If $|r|<1$, then $S_\infty = \frac{u_1}{1-r}$

Derive the formula

$$S_n = u_1 + u_1 r + u_1 r^2 + \dots + u_1 r^{n-1}$$

$$S_n - u_1 = u_1 r + u_1 r^2 + \dots + u_1 r^{n-1}$$

* Mult 1st equation by r *

$$rS_n = u_1 r + u_1 r^2 + u_1 r^3 + \dots + u_1 r^{n-1} + u_1 r^n$$

$$rS_n = S_n - u_1 + u_1 r^n$$

$$rS_n - S_n = -u_1 + u_1 r^n$$

$$S_n(r-1) = -u_1 + u_1 r^n$$

$$S_n = \frac{-u_1 + u_1 r^n}{r-1}$$

$$S_n = \frac{u_1(-1+r^n)}{r-1}\left(\frac{-1}{-1}\right)$$

$$S_n = \frac{u_1(1-r^n)}{-r+1}$$

ex 1: Use the geometric sequence:

$$4, \frac{-4}{3}, \frac{4}{9}, \frac{-4}{27} \ldots$$

a) Find the sum of the first 10 terms

$r = \frac{-1}{3}$

$$S_{10} = \frac{4\left(1-\left(\frac{-1}{3}\right)^{10}\right)}{1-\left(\frac{-1}{3}\right)}$$

$$\boxed{S_{10} = 3}$$

b) If possible, find the sum of ∞ many terms.

$r = \frac{-1}{3}$ ✓

$$S_\infty = \frac{4}{1-\frac{-1}{3}}$$

$$\boxed{S_\infty = 3}$$

ex 2: u_n is a geometric sequence in which $u_3 = 20$ and $u_6 = 160$.

a) Find u_1 and r

$20 = u_1 r^{3-1}$

$20 = u_1 r^2$

$\frac{20}{r^2} = u_1$

$160 = u_1 r^{6-1}$

$160 = u_1 r^5$

$\frac{160}{r^5} = u_1$

$$\frac{20}{r^2} = \frac{160}{r^5}$$

$$20r^5 = 160r^2$$

$$20r^3 = 160$$

$$r^3 = 8$$

$$\boxed{r = 2}$$

$$u_1 = \frac{20}{2^2}$$

$$\boxed{u_1 = 5}$$

b) $$\sum_{n=1}^{12} u_n$$

$$S_{12} = \frac{5(1-2^{12})}{1-2}$$

$$\boxed{S_{12} = 20,475}$$

ex 3: Find the sum

a) $\sum_{n=1}^{20} 3\left(\frac{1}{5}\right)^n$

$$S_{20} = \frac{\frac{3}{5}\left(1-\left(\frac{1}{5}\right)^{20}\right)}{1-\frac{1}{5}}$$

$$\boxed{S_{20} = \frac{3}{4} \text{ or } .75}$$

b) $\sum_{n=1}^{\infty} 9\left(\frac{1}{2}\right)^n$

$$S_\infty = \frac{\frac{9}{2}}{1-\frac{1}{2}}$$

$$\boxed{S_\infty = 9}$$

c) Find y if $\sum_{n=1}^{\infty} \left(\frac{y}{5}\right)^{n-1} = 5$

$$S_\infty = \frac{1}{1-\frac{y}{5}}$$

$$5 = \frac{1}{1-y/5}$$

$$5\left(1-\frac{y}{5}\right) = 1$$

$$5 - y = 1$$

$$-y = -4$$

$$\boxed{y = 4}$$