Unit 8 : Sequences and Series. (Calculator is allowed)

#1.

a) $t_n = 10 + (n-1)(-4) = 14 - 4n$ Arithematic

b) $t_n = (-2)(-3)^{n-1} = \left(\frac{2}{3}\right)(-3)^n$ Geometric

c) $t_n = n^2 + 1$ Neither

#2. a) $t_1 = 3$
$t_n = (t_{n-1})^2$

b) $t_1 = -8$
$t_n = t_{n-1} + 3$

#3. a) 5, 17, 54, 166

b) -2, 1, 6, 12

#4. a) $t_n = t_1 + (n-1)\cdot d \Rightarrow t_{20} = 12 + (19)\cdot 7 = \boxed{145}$

$40 = t_1 + 4\cdot d$
$61 = t_1 + 7\cdot d$
$\Rightarrow \boxed{\begin{matrix} d = 7 \\ t_1 = 12 \end{matrix}}$

b) $t_n = t_1 \cdot r^{n-1}$

$10 = t_1 \cdot r^2$
$160 = t_1 \cdot r^6$
$\Rightarrow \frac{160 = t_1 \cdot r^6}{10 = t_1 \cdot r^2} \Rightarrow 16 = r^4 \Rightarrow r = 2$
$t_1 = \frac{5}{2}$

$t_{12} = \left(\frac{5}{2}\right)(2)^{11} = \boxed{5120}$

#5. a) Arithmetic : $t_1 = 5 \quad d = 3 \Rightarrow \boxed{n = 200}$

$602 = 5 + (n-1)\cdot 3$

$S_{200} = \frac{200\,(5+602)}{2} = \boxed{60,700}$

#5. (b) $t_1 = 1 \quad t_{20} = 115$ Arithmetic.

$$S_{20} = \frac{20(1+115)}{2} = \boxed{1,160}$$

(c) $t_1 = 24 \quad t_n = \frac{3}{256} \quad r = \frac{1}{2}$.

$$\Rightarrow S_n = \frac{24 - \left(\frac{3}{256}\right)\left(\frac{1}{2}\right)}{1 - \frac{1}{2}} = \boxed{\frac{12,285}{256}}$$

#6. (a) Geometric: $r = \frac{-1}{3}, \quad t_1 = 27, \quad n = 10$

$$S_{10} = \frac{27\left(1 - \left(\frac{-1}{3}\right)^{10}\right)}{1 - \frac{1}{3}} = \boxed{\frac{29,524}{729}}$$

(b) Infinite G Series: $S = \frac{t_1}{1-r} = \frac{27}{1-\left(-\frac{1}{3}\right)} = \boxed{\frac{81}{4}}$

#7. 3, 6, 9.... G series: $r = 3 \quad n = 50$

$$S_{50} = \frac{3(1-(3)^{50})}{1-3} = \boxed{1.0768 \times 10^{24}}$$

#8. (a) $72 + 105 + 144 + 189 + 240 = 750$

(b) Infinite G series: $r = \frac{-2}{3} \quad t_1 = (4)\left(\frac{-3}{2}\right)^2 = 9$

$$S = \frac{9}{1-\left(\frac{-2}{3}\right)} = \boxed{\frac{27}{5}}$$

#9. (a) $t_n = (1)\left(\frac{1}{8}\right)^{n-1} = (8)\left(\frac{1}{8}\right)^n$

$$\boxed{\sum_{k=1}^{5} 8\left(\frac{1}{8}\right)^k}$$

(b) $t_n = 100 + (n-1)(-3)$

$= 103 - 3n$

$40 = 103 - 3n$

$n = 21$

$$\boxed{\sum_{k=1}^{21} (103 - 3k)}.$$

#10 (a) ∞ (b) $\frac{3}{4}$ (c) 0

#11. $r = \frac{x}{4}$, $t_1 = x$

(a) $\left|\frac{x}{4}\right| < 1 \Rightarrow$ $-4 < x < 4$

(b) $S = \frac{x}{1 - \frac{x}{4}} = \frac{4x}{4 - x}$

#12. (a) 12 m

(b) $2[2 + 4 + 6 + 8 + 10 + 12] = 84$

(c) $\sum_{k=1}^{n} 4k$

(d) $t_n = 2n = 50 \quad n = 25 \Rightarrow$ 26 children

#13. $t_1 = 12{,}000$

$t_n = 12000(.85)^n$

(a) Decay factor : 0.85

(b) $t_n = (t_{n-1})(.85) \quad t_1 = 12{,}000$

(c) $t_n = 12000(.85)^n$

$12000(0.85)^n = 3000$

$n \cdot \ln(0.85) = \ln\left(\frac{1}{4}\right)$

$n = \frac{\ln\left(\frac{1}{4}\right)}{\ln(0.85)}$

≈ 8.75 yrs

It will take 9 yrs