CHAPTER 1 CONCEPTS

- Completing the Square
- Discriminant
- Sum and Product of Roots
- Quadratic Formula
- Maximum and Minimum
- Solving Equations
- Problem Solving
- Graphing
- Optimization

- In a Quadratic Function ax^2+bx+c, the sum of the roots is $-\frac{b}{a}$ and the product of the roots is $\frac{c}{a}$
- The discriminant is b^2-4ac

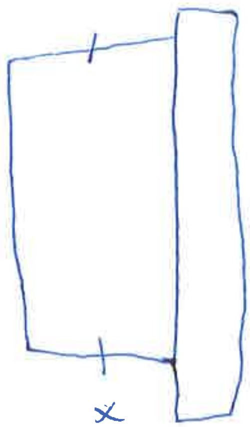
  If there are:

  No real roots: $b^2-4ac<0$

  One real root: $b^2-4ac=0$

  Two real roots: $b^2-4ac>0$

1.



40m of fencing, one side is brick wall, find max area.

$x \cdot (40-2x) = 40x - 2x^2$

$-2x^2+40x = A$

$-2x^2+40x=0$

$-2(x^2-20x)$

$-2x(x-20)=0$

$x=0 \quad x=20$

$x=10$ is midway

$-2(10)^2+40(10)=A$

$-200+400=A$

$200=A$

$10, 200$

$\boxed{x=10m \quad y=20m}$

2. Solve using the quadratic formula.

$x^2+5x+3=0$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-5 \pm \sqrt{25-4(1)(3)}}{2}$$

$$\boxed{x = \frac{-5 \pm \sqrt{13}}{2}}$$

96 miles ~~mph~~ - ~~Patrick~~ If he can ride 4 mph faster, he can complete the same distance in 2 hours less than usual. What is Patrick's usual speed

x y

Distance = Speed · time

$96 = x+4 \cdot y-2$

$96 = xy$

$96 = (x+4)(\frac{96}{x} - 2)$

$96 = 96 - 2x + \frac{384}{x} - 8$

$96 = 96$

$+8$

$104 - 96 = -2x + \frac{384}{x}$

$8 = -2x + \frac{384}{x}$

$8x = -2x^2 + 384$

$2x^2 - 8x + 384 = 0$

$-2(x^2 + 4x - 192) = 0$

$2(x^2 - 4x + 192) = 0$

$2(x^2 - 4x + 192)$

$2(x+16)(x-12) = 0$

$x = -16$

$2(x+16)(x-12) = 0$

$\boxed{12 \text{mph}}$

$-16 - 12$

$96 - 2x + \frac{384}{x} - 8 = 96$

$144 + 48$

$-2x + \frac{384}{x} - 8 = 0$

196

$-2x^2 - 8x + 384 = 0$

$2x^2 + 8x - 384 = 0$

$2(x^2 + 4x - 192) = 0$

$2(x-12)(x+16) = 0$

$x = 12$

~~$x = -16$~~

$x = 12$

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Chapter 2: Functions - Concepts

1. A function has no two ordered pairs with the same x-value. A relation is any set of points which connect two variables.

2. Inverse function is a reflection of a regular function over the axis $y=x$. In this case, the values of domain and range are switched.

3. Sign Diagram - represents the x-axis of an equation's graph.
   - includes the x-ints as marks on the line.

The positivity of the graph is determined by testing values in-between the roots on the line.

4. Composite functions are where one function is put into another one.

   Example: $(f \circ g)(x)$

5. Domain and Range - Domain: the set of possible values on the Horizontal Axis (Input)

   Range: the set of possible values on the Vertical Axis (Output)

6. Modulus Functions & Inequalities
   - Solved by squaring both sides $|x+a|^2 = |x+c|^2$
   - Algebraically by the sign diagram and graphically by entering the equation into a graphing calculator.

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Chapter 2: Functions - Examples

1.) If $f(x) = -2x$ and $g(x) = \dfrac{x}{x^2-3x+2}$, find $(f \circ g)(x)$ and its domain.

$$(f \circ g)(x) = -2\left(\frac{x}{x^2-3x+2}\right)$$

$$= \frac{-2x}{(x-2)(x-1)} \qquad x \neq 2,\ x \neq 1$$

Domain: $x \in (-\infty, 1) \cup (1,2) \cup (2, \infty)$

2.) Find the equation of $h^{-1}(x)$ if $h(x) = \sqrt{x-2}+1$.

$$x = \sqrt{h^{-1}(x)-2}+1$$

$$h^{-1}(x) = (x-1)^2+2$$

$$h^{-1}(x) = x^2-2x+3$$

3.) Solve $|x+1| = |2x-3|$

$$|x+1|^2 = |2x-3|^2$$

$$x^2+2x+1 = 4x^2-12x+9$$

$$3x^2-14x-8=0$$

$$(3x-2)(x-4)=0$$

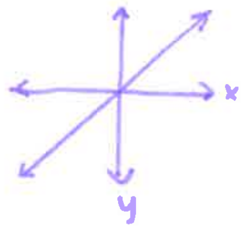
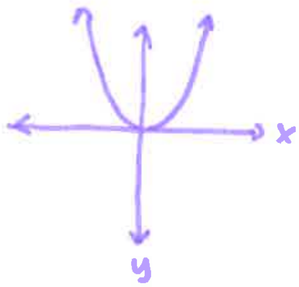
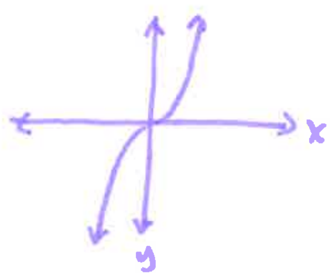
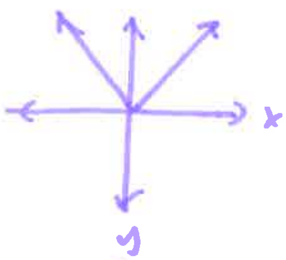
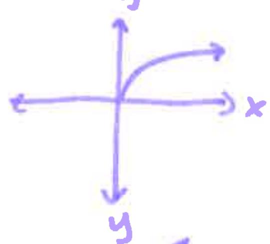
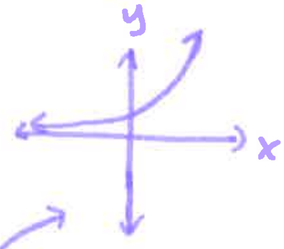
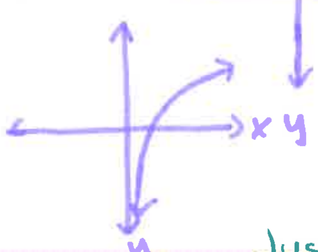
$$\begin{cases} x = \frac{2}{3} \\ x = 4 \end{cases}$$

CHAPTER 5

transformations:

Summary:

- transformations from parent functions
- $y = p \cdot f(q(x-a)) + b$

PARENT NAME:	PARENT EQUATION:	SKETCH:	KEY POINTS:	DOMAIN/RANGE:
• Linear	$f(x)=x$		x-int: (0,0) y-int: (0,0)	D: $x \in \mathbb{R}$ R: $y \in \mathbb{R}$
• Quadratic	$f(x)=x^2$		vertex: (0,0)	D: $x \in \mathbb{R}$ R: $y \in [0,\infty)$
• Cubic	$f(x)=x^3$		x-int: (0,0) y-int: (0,0)	D: $x \in \mathbb{R}$ R: $y \in \mathbb{R}$
• Modulus	$f(x)=\|x\|$		vertex: (0,0)	D: $x \in \mathbb{R}$ R: $y \in [0,\infty)$
• Square Root	$f(x)=\sqrt{x}$		x-int: (0,0) y-int: (0,0)	D: $x \in [0,\infty)$ R: $y \in [0,\infty)$
• Exponential	$f(x)=2^x$		x-int: NEVER HA: y=0	D: $x \in \mathbb{R}$ R: $y \in (0,\infty)$
• Log	$f(x)=\log_2 x$		x-int: (1,0) VA: x=0	D: $x \in (0,\infty)$ R: $y \in \mathbb{R}$

Justin K, Daven K, Samantha P, Stephanie X

Group 3 - Chapter 3 Period 5

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Summary

- Exponent Rules
  - addition, subtraction, multiplication, division
- Algebraic Expansion
  - Quadratic Exponential Equations
- Factorization
- Graphing
  - Exponents
    - → 2^x
    - → e^x
    - → etc.
- Solving Equations
  - Finding x in an exponential equation
- The Natural Base e
  - Know what it is
  - How to use it

1. Simplify

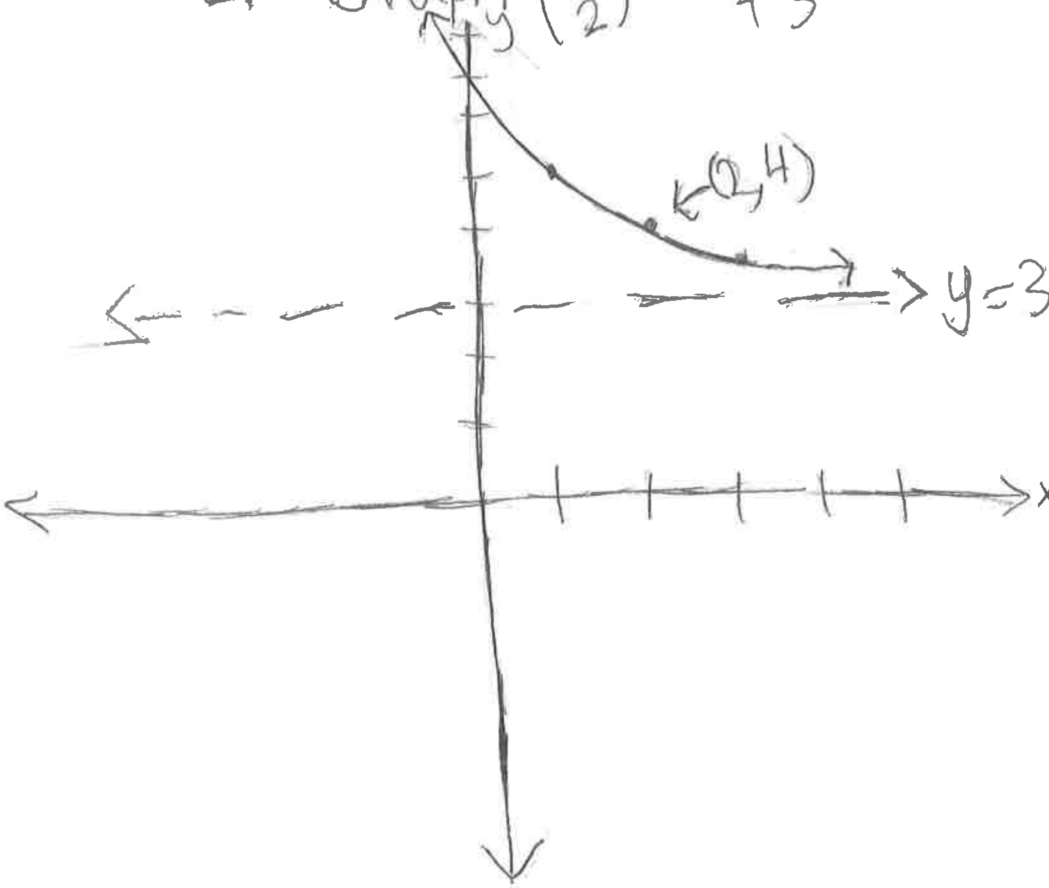
$$\frac{20^n - 4^{n+2}}{4^n}$$

$$\frac{\cancel{4^n} \cdot 5^n - \cancel{4^n} \cdot 4^2}{\cancel{4^n}}$$

$5^n - 4^2$

$\boxed{5^n - 16}$

3. Graph $\left(\frac{1}{2}\right)^{x-2} + 3$



2. $7(7^x) + 7(7^{-x}) = 50$

$7^x \cdot \left(7 \cdot 7^x + \frac{7}{7^x}\right) = 50 \cdot 7^x$

$7 \cdot 7^{x^2} + 7 = 50 \cdot 7^x$

$7 \cdot A^2 + 7 = 50A$

$7A^2 - 50A + 7 = 0$

$7A^2 - 49A - A + 7 = 0$

$7A(A-7) - 1(A-7) = 0$

$(7A-1)(A-7) = 0$

$A = \frac{1}{7} \quad A = 7$

$7^x = \frac{1}{7} \quad A^x = 7$

$x = -1$	$x = 1$

Laws of Logarithms

$\log_x a + \log_x b = \log_x ab$

$\log_x a - \log_x b = \log_x \frac{a}{b}$

$b \log_x a = \log_x a^b$

$\log_b a = \frac{\log a}{\log b} = \frac{\ln a}{\ln b} = \frac{\log_x a}{\log_x b}$

$x^{\log_x a} = a$

Definition of a Log

$\log_b x = y \longleftrightarrow b^y = x$

★ x can never be less than or equal to 0 ★

Growth and Decay

If $1<b$, it is a growth.

if $0<b<1$ it is a decay

Chapter 4

① $\log_4 64 + \log_9 \frac{1}{81} - \log_{10} 1 = \log_5 x$

$\log_4 4^3 + \log_9 9^{-2} - \log_{10} 10^0 = \log_5 x$

$3 - 2 - 0 = \log_5 x$

$\log_5 x = 1$

$5^1 = x$

$x = 5$

② write as a single log

$\log_{\frac{1}{3}} x - \log_{27} y + \frac{1}{3} \log_9 z$

$\frac{\log x}{\log \frac{1}{3}} - \frac{\log y}{\log 27} + \log_9 z^{\frac{1}{3}}$

$\frac{\log x}{\log 3^{-1}} - \frac{\log y}{\log 3^3} + \frac{\log \sqrt[3]{z}}{\log 3^2}$

$\frac{\log x}{-\log 3} - \frac{\log y}{3 \log 3} + \frac{\log \sqrt[3]{z}}{2 \log 3}$

$\frac{-6 \log x}{6 \log 3} - \frac{2 \log y}{6 \log 3} + \frac{3 \log \sqrt[3]{z}}{6 \log 3}$

$\frac{\log x^{-6} - \log y^2 + \log (\sqrt[3]{z})^3}{6 \log 3}$

$\frac{\log \frac{x^{-6} z}{y^2}}{\log 3^6}$

$\frac{\log \frac{z}{x^6 y^2}}{\log 729}$

$\log_{729} \frac{z}{x^6 y^2}$

Dimitri K.
Marius R.
Trisha B.
Pranita R.

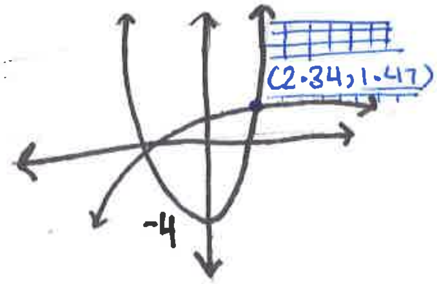
Use your graphing calculator to solve each equation or inequality

$x^2 - 4 > \ln(x+2)$

$y > \ln(x+2)$

$x^2 - 4 > y$

$(2.34, 1.47)$



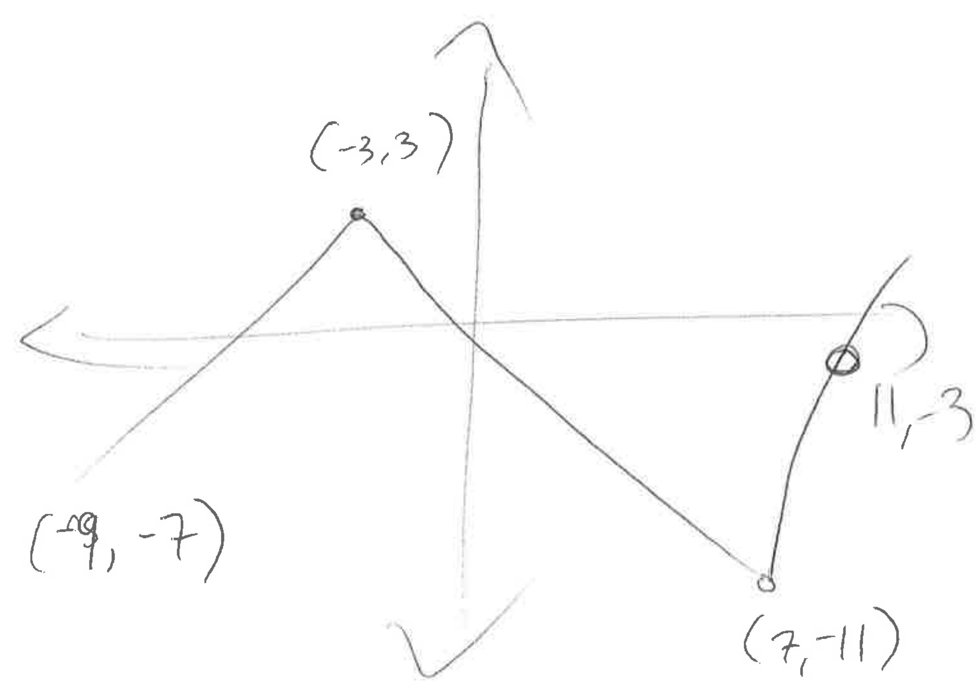
Trisha D., Pranita R., Marius R., Dimitri K.

Chapter 5

1) Use the graph of the function f. Sketch the graph of $k(x) = -2f\left(\frac{1}{2}(x+1)\right) - 1$. Be clear of the key point for $f(x)$ and where they are on $k(x)$.

$2x-1$	x	y	$-2y-1$
-9	-4	3	-7
-3	-1	-2	3
7	4	5	-11
11	6	1	-3

- Reflect across x-axis
- Vertical dilation by -2
- horizontal dilation by 2
- translate left 1
- translate down 1



p: vertical dilation

q: horizontal dilation

a: translation on x-axis

b: translation on y-axis

$f(-x)$: reflected over y-axis

$-f(x)$: reflected over x-axis

2) The function $y=f(x)$ is transformed to the function $g(x)=-f(3(x-2))+5$.

a) List the order of the transformations.

- reflect across x-axis
- horizontal dilation by $\frac{1}{3}$
- translate right 2
- translate up 5

b) Given that $(8,-3)$ lies on $f(x)$, find the coordinates of the corresponding point on $g(x)$.

$\frac{1}{3}x+2$	x	y	$-y+5$
$\frac{14}{3}$	8	-3	8

$-(-3)+5=8$

$\frac{1}{3}(8)+2=\frac{14}{3}$

$\boxed{\left(\frac{14}{3}, 8\right)}$

Chapter 6

Complex Numbers

$\frac{1+3i}{2-5i}$

$= \frac{1+3i}{2-5i}\left(\frac{2+5i}{2+5i}\right) \leftarrow$ conjugates

$= \frac{2+11i+15i^2}{4-25i^2} = \frac{-13+11i}{29}$

* never leave i in the denominator

$= \frac{-13}{29} + \frac{11i}{29}$

Real Polynomials

$(x+yi)(2-i) = -i$

$2x - xi + 2yi - yi^2 = -i$

$2x - xi + 2yi + y + i = 0$

$2x + y = 0$

$-x + 2y = -1$

$y = -\frac{2}{5}$ $x = \frac{1}{5}$

Zeros, Roots, & Factors

find a cubic polynomial in standard form that has zeros $\frac{2}{3}$, $4 \pm i$

$P(x) = (x-\frac{2}{3})(x-(4+i))(x-(4-i))$

$= (x-\frac{2}{3})(x-4-i)(x-4+i)$

$= (x-\frac{2}{3})((x-4)-i)((x-4)+i)$

$= (3x-2)(x^2-8x+17)$

$(x-4)^2-(i)^2$

$= x^2-8x+16+1$

$= x^2-8x+17$

$= x^3-8x^2+17x-\frac{2}{3}x^2+\frac{16}{3}x-\frac{34}{3}$

$= x^3-\frac{26}{3}x^2+\frac{67}{3}x-\frac{34}{3}$

Synthetic Division

$\frac{6x^4+23x^3-6x^2-53x+30}{2x+5}$

$2x+5=0$

$x = -\frac{5}{2}$

$$\begin{array}{r|rrrrr} -\frac{5}{2} & 6 & 23 & -6 & -53 & 30 \\ & \downarrow & -15 & -40 & 115 & -155 \\ \hline & 6 & 8 & -46 & 62 & -125 \end{array}$$

$\Rightarrow 6x^3+8x^2-46x+62$ R -125

Find all quartic Polynomials w/ real coellicent that have zeros 4 and $2-3i$

$(x-4)^2(x-(2-3i))(x-(2+3i))$

$= (x^2-8x+16)(x^2-4x+13)$

$= (x-2)^2-(3i)^2$

$= x^2-4x+4+9$

$= x^2-4x+13$

Long Division

If $P(x) = c_1x^n + c_2x^{n-1} + c_3x^{n-2} + R$ is divided by $(ax+b)$,

$\frac{P(x)}{ax+b} = Q(x) + \frac{R}{ax+b} \leftarrow$ Remainder

where $(ax+b)$ is the divisor, $Q(x)$

↑

quotient

The Remainder Theorem

When $P(x)$ is divided by $x-a$, the remainder is $P(a)$

The Factor Theorm

: k is a zero of $P(x)$ if and only if $x-k$ is a factor of $P(x)$

The Fundamental Theorem of Algebra

Every polynomial of degree n has exactly n roots (some may be real or imaginary)

Liz Huang, Iralde Sicilia, Max Chu, Pranav A.

- Roots & zeros

x^2-x+2 → 1st, factor it,

$(x+1)(x-2)$ → Now because of the zero product property, to find where $y=0$, we need to set $x=$ to the constant of each factor to make the factors $=0$. the factors are -2 & 1, so $x=-1$ & $x=2$.

- End behavior
  - Depends on degrees & coefficients

$2x^2+4x-2$

Even degree means the end behavior is either ↑↑ or ↓↓

Positive coefficient says that the E.B. is either ↓↑ or ↑↑, and that the last arrow is ↑

Consistent one is correct. This is ↑↑

$-3x^5+8x-3$

Odd degree is ↑↓ or ↓↑

Positive coefficient is ↑↓ or ↓↓ (last arrow ↓)

Consistent one is correct

CHAPTER 6: POLYNOMIALS

MULTIPLICITY — the number of times a given root appears in a polynomial equation.

Ex: $(x-2)^2 \rightarrow$ multiplicity $=2$ $\quad (x-2)(x-1) \rightarrow$ multiplicity $=1$

END BEHAVIOR — what happens to y when x approaches ∞ or $-\infty$.

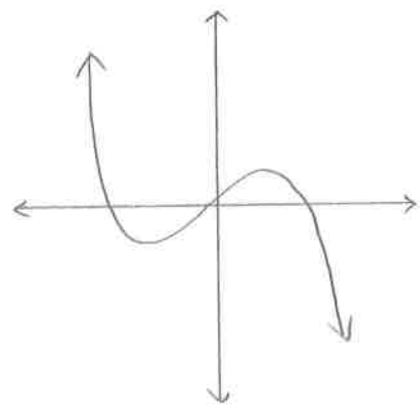
Ex:

	Even	Odd
+	$\nwarrow \nearrow$	$\swarrow \nearrow$
−	$\swarrow \searrow$	$\nwarrow \searrow$

$-4x^3+2x^2+3x+10$

$\downarrow \quad \downarrow$

Neg. Odd



ROOTS AND ZEROS — the x-values at which $y=0$.

Ex: $(x-(1+3i))(x-(1-3i))(x+3)^2(2x-1) \rightarrow$ Roots: $x = 1+3i, 1-3i, -3, \frac{1}{2}$

FACTOR THEOREM — if k is a zero of $P(x)$, then $(x-k)$ if a factor of $P(x)$, and vice versa.

RATIONAL ROOT THEOREM — the solution to any given polynomial where $P(x)=0$ is a root.

Ex:

$x^2+2x+1=0 \rightarrow$ Root: $x=-1$

SUM AND PRODUCT OF ROOTS — if the greatest exponent in an equation is odd, the sum is $-\frac{b}{a}$ and the product is $-\frac{c}{a}$; if the greatest exponent is even, then the sum is $-\frac{b}{a}$ and the product is $\frac{c}{a}$.

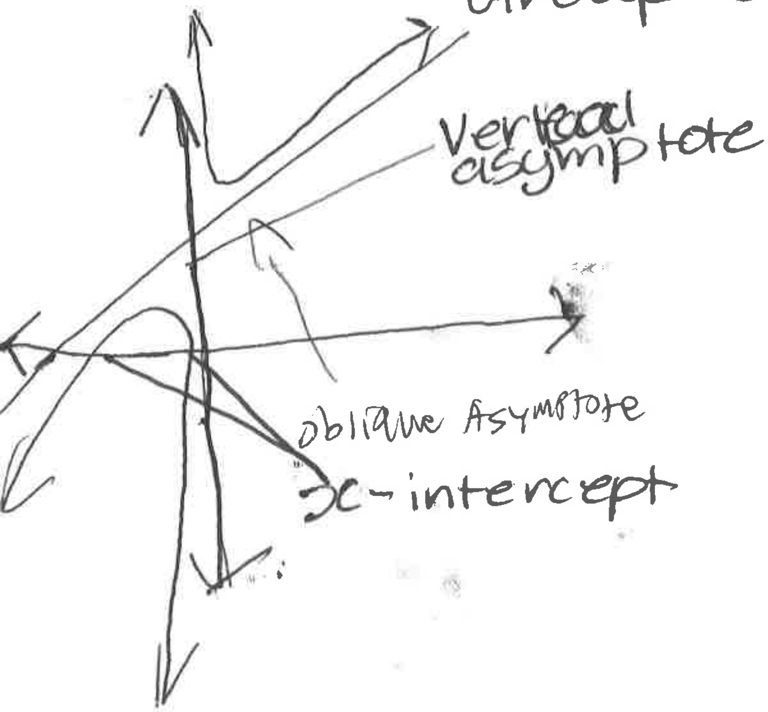
Ex: $x^2-4x+5 \rightarrow$ Sum $= -\frac{-4}{1} \Rightarrow 4$ $\quad 4x^2+2x-6 \rightarrow$ Sum $= -\frac{2}{4} \Rightarrow -\frac{1}{2}$

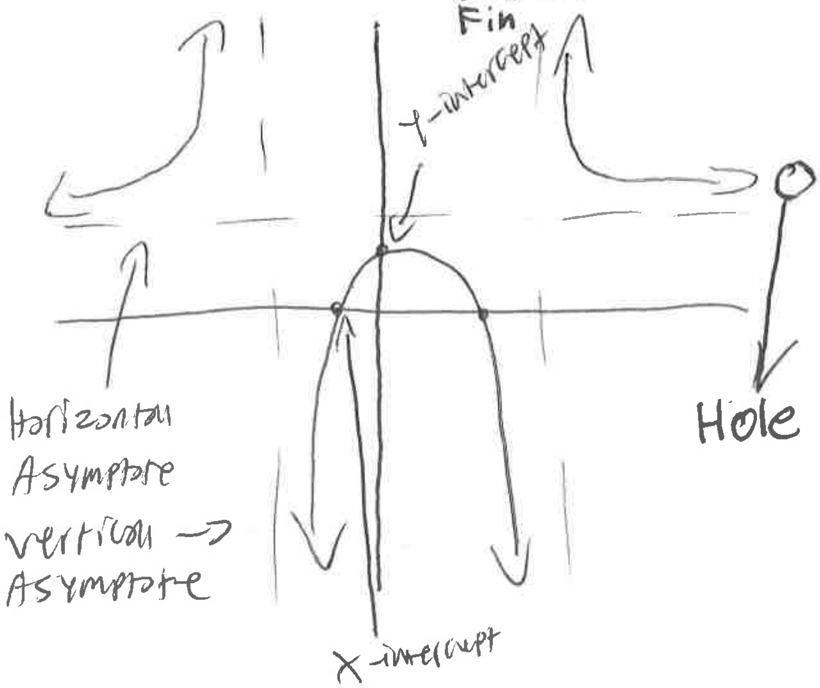
Product $= \frac{5}{1} \Rightarrow 5$ $\quad$ Product $= \frac{-6}{4} \Rightarrow -\frac{3}{2}$

Rational Functions

Group 8

Vatsal chander
Kevin
Varon
Fin





$$\frac{ax+b}{cx^2+dx+e}$$

Horizontal asymptote
$y=0$

$$\frac{ax^2+bx+c}{dx^2+ex+f}$$

Horizontal Asymptote
$y=\frac{a}{d}$

$$\frac{ax^2+bx+c}{dx+e}$$

Horizontal Asymptote
$y=$ none

oblique asymptote, to find, divide

How to find x-int: set y to zero

How to find y-int: set x to zero

How to find holes: Any factored terms that cancel out give the x-coord of the hole, plug back into simplified to find y-coord

How to find vertical asymptote: any values that make denominator zero

Inequalities

$$\frac{x-1}{x+2} > 3$$

$$\downarrow$$

$$\frac{x-1}{x+2} - 3 > 0$$

$$\frac{x-1-3(x+2)}{x+2} > 0$$

$$\frac{x-1-3x-6}{x+2} > 0$$

$$\frac{-2x-7}{x+2} > 0 \longrightarrow -2x-7=0$$

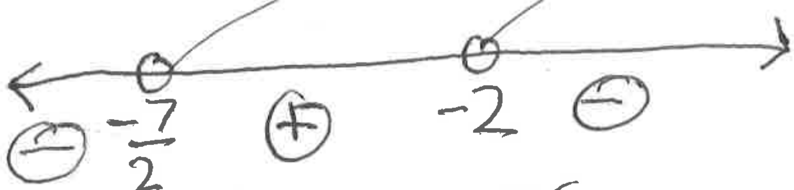
$$x = \frac{-7}{2}$$

$$\downarrow$$

$$x+2=0$$

$$x=-2$$

$$\left(-2, \frac{-7}{2}\right)$$



1) Solve for x
2). Put on sign diagram.

Modulous Inequalities

$$\left|\frac{x+2}{x+3}\right| \geq 0$$

$[-3, -2]$

$$\frac{x+2}{x+3} \geq 0$$

$x = -3$

$x = -2$

$$\frac{x+2}{x+3} \leq 0$$

$x = -2$

$x = -3$

Summary

1). Take original function and make

2, fliping the inequality and changing

the sign of the other side.

2). Solve for x

3) numerator and denominator are points on diagram.

4) if ($\geq$) or ($\leq$) then include numerator.