IB Math HL 1 21A Area under a Curve Name

Goal: Estimate the area under a curve using rectangle approximations.

1. The graph of  is shown. We are going to approximate the area under this function on the interval .

a. Six rectangles have been drawn. What is the width of each rectangle?

b. These rectangles are called left-hand rectangles because the height of each rectangle is determined by the function’s height at the left x-value of the interval. For example, the height of the rectangle at  is 7. Approximate the area under the curve by finding the sum of the areas of all six rectangles.



c. Is the approximation from part b too high or too low? How can you tell?

d. Sketch 6 right-hand rectangles and compute this approximate area.

2. Let’s approximate the area under the curve using 30 left-hand rectangles!

a. How wide will each rectangle be?

b. If we are using 30 left-rectangles, explain why the area of the first rectangle can be written as .

c. Explain why the total area of all 30 left-rectangles can be written as



d. Use your graphing calculator to evaluate the sigma expression in part c.

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|  | **TI-84** |  | **TI-Nspire** |
|  | Enter the function using Y =  QUIT to home screen  LIST, MATH, 5:SUM  LIST, OPS, 5:SEQ  Make it look like this:  sum(seq(.1Y1(2+.1K), K, 0, 29, 1))  To get Y1, VARS, Y-VARS, 1, 1 | You may need to fill in blanks like this:  Exp: Y1(K)  Variable: K  Start: 2  End: 5  Step: 1 | Add a new Calculator page  Define the function like this:  f(x):=11-2x  above button next to 9  Menu, 4:Calculus, 3:Sum  Make it look like the sigma notation |

3. Now let’s approximate the area under the curve using 100 right-hand rectangles!

a. Complete the sigma notation: 

b. Use your graphing calculator to evaluate the sigma expression in part b.

c. Compare this result to that of #2. Which is more accurate? Why?



4. a. Use your results to estimate the actual area under 

on the interval .

b. Use your knowledge of geometry to calculate the actual area.

5. Approximate the area under  on the interval .

a. Sketch  on the interval .

b. Will left or right rectangles give a lower estimate of the area?

c. Using the indicated number of rectangles, write in the sigma notation and approximate the upper and lower areas.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Number of rectangles | Lower Sigma Notation | Lower Area | Upper Sigma Notation | Upper Area |
| 8 |  |  |  |  |
| 50 |  |  |  |  |
| 1000 |  |  |  |  |

d. Based on your results, estimate the exact area of this region.