IB Pre HL 24E Compound Events

1. A six-sided die is rolled and a coin is tossed.

P(H) = $\frac{1}{2}$

P(3) = $\frac{1}{6}$

P(H and 3) = $\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$

Independent Events:
One event does not affect the probability of the other.

A and B are independent if and only if P(A and B) = $P(A) \cdot P(B)$

2. A bag has 4 Blue, 5 Green, 2 Red, and 9 Yellow marbles.
Two marbles are drawn.

Without replacement
P(B, then G) (Dependent Events)

$P(B) = \frac{1}{5}$

$P(B \cdot G) = \frac{1}{5} \cdot \frac{5}{19} = \boxed{\frac{1}{19}}$

With replacement
P(B, then G) (Independent Events).

$P(B) = \frac{1}{5}$

$P(B \cdot G) = \frac{1}{5} \cdot \frac{1}{4} = \boxed{\frac{1}{20}}$

Dependent Events:
The probability of the 2nd event depends on the 1st outcome.

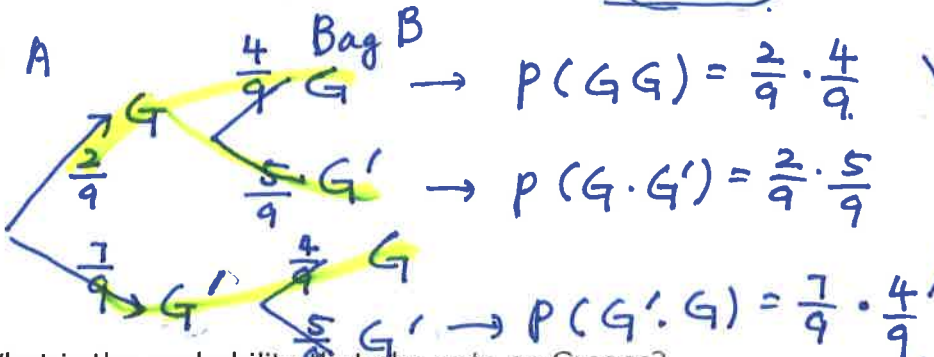
P(A, then B) = $P(A) \cdot P(B,$ Given that A has occured$)$

24F Tree Diagrams

3. Holly has two bags of marbles:
   Bag A has 3 Red, 2 Green, and 4 Blue ⇒ 9 marbles
   Bag B has 2 Red, 4 Green, and 3 Blue ⇒ 9 marbles

Holly randomly selects a marble from each bag.

a. What is the probability that she gets at least one Green?



$P(GG) = \frac{2}{9} \cdot \frac{4}{9}$

$P(G \cdot G') = \frac{2}{9} \cdot \frac{5}{9}$

$P(G' \cdot G) = \frac{7}{9} \cdot \frac{4}{9}$

$\frac{8}{81} + \frac{10}{81} + \frac{28}{81} = \boxed{\frac{46}{81}}$

b. What is the probability that she gets no Greens?

$P(G'G') = \frac{7}{9} \cdot \frac{5}{9} = \boxed{\frac{35}{81}}$

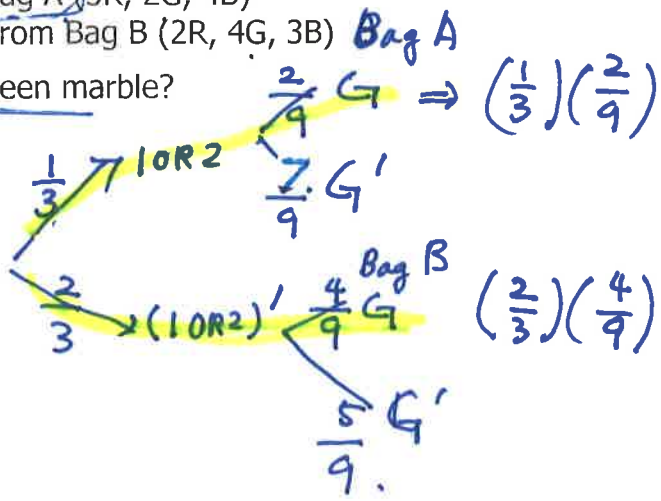
OR $= 1 - \frac{46}{81} = \boxed{\frac{35}{81}}$

4. Carolyn has a six-sided die. If she rolls a...
   ...1 or 2, Holly draws a marble from Bag A (3R, 2G, 4B)
   ...3, 4, 5, or 6, Holly draws a marble from Bag B (2R, 4G, 3B)

What is the probability that Holly draws a Green marble?

$P(1 \text{ OR } 2) = \frac{2}{6} = \frac{1}{3}$

$P(3, 4, 5, \text{OR } 6) = \frac{2}{3}$



$\left(\frac{1}{3}\right)\left(\frac{2}{9}\right)$

$\left(\frac{2}{3}\right)\left(\frac{4}{9}\right)$

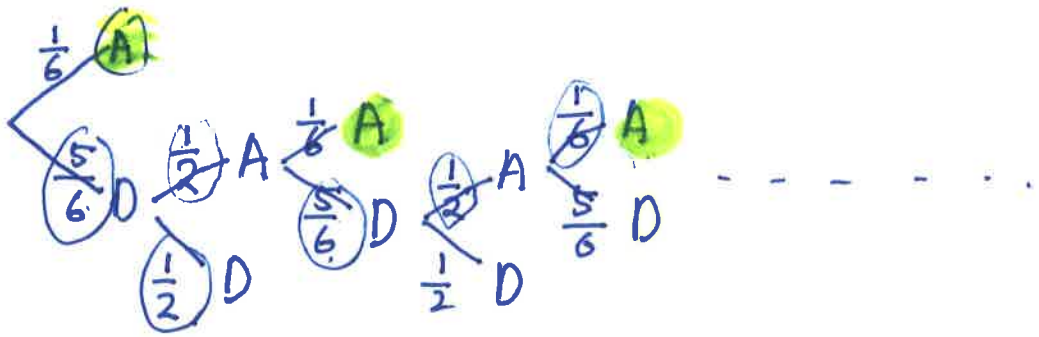
$\Rightarrow P(G) = \left(\frac{1}{3}\right)\left(\frac{2}{9}\right) + \left(\frac{2}{3}\right)\left(\frac{4}{9}\right)$

$= \boxed{\frac{10}{27}}$

2, 4, 6

5. Adam and Daniel are taking a math test on probability and randomly decide to take a break and play a game of dice. Adam begins the game by rolling a single die, and wins if he rolls a 5. If he doesn't win on his turn, he passes the die to Daniel, who wins if he rolls an even number and passes the die back to Adam if he doesn't win on his turn. The game continues in this fashion until somebody wins.

a) What is the probability that Adam wins the game on his *nth* roll?



$\frac{1}{6}, \left(\frac{5}{6}\cdot\frac{1}{2}\right)\cdot\frac{1}{6}, \left(\frac{5}{6}\cdot\frac{1}{2}\right)\cdot\left(\frac{5}{6}\cdot\frac{1}{2}\right)\cdot\frac{1}{6}, \ldots$

b) What is the probability that Adam wins the game?

$\Rightarrow \frac{1}{6}, \frac{1}{6}\left(\frac{5}{12}\right)^1, \frac{1}{6}\left(\frac{5}{12}\right)^2, \frac{1}{6}\left(\frac{5}{12}\right)^3$

1st 2nd 3rd 4th

$\Rightarrow$ nth Roll $= \left(\frac{1}{6}\right)\left(\frac{5}{12}\right)^{n-1}$

$P(\text{Adam wins}) = \frac{1}{6} + \frac{1}{6}\left(\frac{5}{12}\right) + \frac{1}{6}\left(\frac{5}{12}\right)^2 + \frac{1}{6}\left(\frac{5}{12}\right)^3 \cdots \cdots \infty$

(Geometric) $S_\infty = \frac{a_1}{1-r}$ $\quad a_1 = \frac{1}{6}$ $\quad r = \frac{5}{12}$

$S_\infty = P(\text{Adam wins}) = \frac{\frac{1}{6}}{1-\frac{5}{12}} = \frac{\frac{1}{6}}{\frac{12}{12}-\frac{5}{12}} = \frac{\frac{1}{6}}{\frac{7}{12}}$

$= \left(\frac{1}{6}\right)\cdot\frac{12^2}{7} = \boxed{\frac{2}{7}}$