1. Find a cubic polynomial in standard form that has zeros $\frac{2}{3}$, $4 \pm i$.

$$P(x) = \left(x - \frac{2}{3}\right)\left(x - (4+i)\right)\left(x - (4-i)\right)$$

$$= \left(x - \frac{2}{3}\right)\left[(x-4) - i\right]\left[(x-4) + i\right]$$

$$= \left(x - \frac{2}{3}\right)\left[(x-4)^2 - i^2\right]$$

$$= \left(x - \frac{2}{3}\right)\left[x^2 - 8x + 17\right] \Rightarrow (3x-2)(x^2 - 8x + 17)$$

$$= 3x^3 - 24x^2 + 51x - 2x^2 + 16x - 34$$

$$= \boxed{3x^3 - 26x^2 + 67x - 34}$$

2. Find all quartic polynomials with real coefficients that have zeros 4 and $2 - 3i$.

double $2 + 3i$

zeros.

$$P(x) = (x-4)^2 \left(x - (2-3i)\right)\left(x - (2+3i)\right)$$

$$= A(x-4)^2\left[(x-2) + 3i\right]\left[(x-2) - 3i\right]$$

$$= A(x^2 - 8x + 16)\left[x^2 - 4x + 4 + 9\right]$$

$$= A(x^2 - 8x + 16)(x^2 - 4x + 13) \qquad A \in \mathbb{R}$$

$$= A(x^4 - 12x^3 + 61x^2 - 168x + 208)$$

Warm Up

1. Find constants a, b, and c given that
$6x^3 + 7x^2 - 19x + 7 = (2x-1)(ax^2 + bx + c)$ for all x.

$$ax^2 + bx + c = \frac{6x^3 + 7x^2 - 19x + 7}{2x - 1}.$$

$$= 3x^2 + 5x - 7$$

$$\therefore \quad a = 3, \quad b = 5, \quad c = -7$$

$$\frac{1}{2} \left| \begin{array}{cccc} 6 & 7 & -19 & 7 \\ & 3 & 5 & -7 \\ \hline 6 & 10 & -14 & 0 \end{array} \right.$$

$$\Rightarrow 3x^2 + 5x - 7$$

2. Find constants a and b if
$z^4 + 9 = (z^2 + az + 3)(z^2 + bz + 3)$

$$\text{See attached.}$$

Example 1) $(x+3)$ is a factor of $p(x) = x^3 + ax^2 - 7x + 6$. Find a where $a \in R$ and the other factors.

$$\frac{x^3 + ax^2 - 7x + 6}{x + 3} \Rightarrow \text{Remainder} = 0.$$

$$-3 \left| \begin{array}{cccc} 1 & a & -7 & 6 \\ & -3 & 9-3a & 9a-6 \\ \hline 1 & a-3 & 2-3a & 9a \end{array} \right.$$

$$9a = 0$$

$$\boxed{a = 0}$$

Example 2) $(2x+3)$ and $(x-1)$ are factors of $2x^4 + ax^3 - 3x^2 + bx + 3$. Find constants a and b and all zeros of the polynomial.

$$z^4 + 9 = z^4 + bz^3 + 3z^2 + az^3 + abz^2 + 3az + 3z^2 + 3bz + 9$$

$$0 = (a+b)z^3 + (6+ab)z^2 + (3a+3b)z$$

$a+b=0$ $6+ab=0$ ~~$3a+3b=0$~~

$\quad a=-b$ $ab=-6$

$\qquad\qquad\qquad -b^2 = -6$

$\qquad\qquad\qquad\quad b^2 = 6$

$\qquad\qquad\qquad\quad b = \pm\sqrt{6}$

$\boxed{\begin{array}{l} a=-\sqrt{6}, \; b=\sqrt{6} \\ a=\sqrt{6}, \; b=-\sqrt{6} \end{array}}$