

#1.

$$e^x = X$$

$$(X-2)(X-3) \leq 2X$$

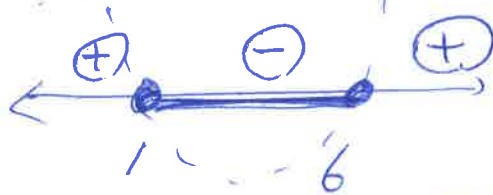
$$X^2 - 5X + 6 \leq 2X$$

$-2X \quad -2X$

$$\Rightarrow X^2 - 7X + 6 \leq 0$$

$$\Rightarrow (X-6)(X-1) \leq 0$$

Sign diagram



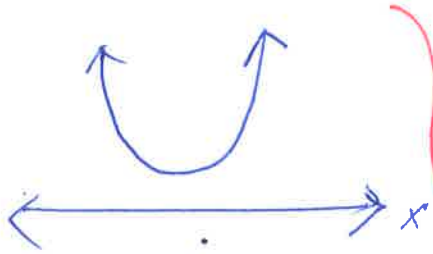
$$X \in [1, 6]$$

$$X \in [0, \ln 6]$$

$$e^x = X = 1 \Rightarrow x = 0$$

$$e^x = X = 6 \Rightarrow x = \ln 6$$

#2. $3x^2 - kx + 12 = y > 0$



$$\Delta = b^2 - 4ac < 0$$

$$a=3 \quad b=-k \quad c=12$$

$$(-k)^2 - (4)(3)(12) < 0$$

$$k^2 - 144 < 0$$

$$(k+12)(k-12) < 0$$

Sign diagram



$$k \in (-12, 12)$$

$$\#3. \quad mx^2 - mx + 1 = 0$$

$$\Delta = b^2 - 4ac = 0$$

$$a = m$$

$$b = -m$$

$$c = 1$$

$$\Rightarrow (-m)^2 - (4)(m)(1) = 0$$

$$\Rightarrow m^2 - 4m = 0$$

$$m(m-4) = 0$$

$$\cancel{m=0} \text{ OR } m=4.$$

$$\text{check } m=0 \Rightarrow 1 \neq 0$$

#4

$$x = \sqrt{x+5} - 3$$

$$\hookrightarrow (x+3)^2 = (\sqrt{x+5})^2$$

$$x^2 + 6x + 9 = x + 5$$

$$\hookrightarrow x^2 + 5x + 4 = 0$$

$$(x+4)(x+1) = 0$$

$$x = -4$$

$$x = -1$$

check

$$x + 3 = \sqrt{x+5}$$

$$-4 = \sqrt{1}$$

$$2 = \sqrt{4}$$

$$\#5 \quad 2|x+3| \leq x+15$$

$$\hookrightarrow |x+3| \leq \frac{x+15}{2}$$

$$\textcircled{1} \quad 2(x+3) \leq \frac{x+15}{2} \quad \text{and} \quad x+3 \geq -\left(\frac{x+15}{2}\right)$$

$$2x+6 \leq x+15$$

$$2x+6 \geq -x-15$$

$$x \leq 9$$

$$3x \geq -21$$

$$x \geq -7$$



$$x \in [-7, 9]$$

$$\#6. \quad y = 5x^2 + 42x + 2 = a.$$

$$\Rightarrow \underline{5x^2 + 42x + 2 - a = 0}$$

5

$$\Rightarrow 0x^2 + \frac{42}{5}x + \left(\frac{2-a}{5}\right) = 0$$

n: even

$$\text{Sum: } -\frac{42}{5}$$

$$\text{product: } \frac{2-a}{5}$$

Notes:

n: odd:

$$k_1 + k_2 + k_1 k_2 = 0$$

$$\frac{-42}{5} + \left(\frac{2-a}{5}\right) = 0$$

$$-42 + 2 - a = 0$$

$$a = -40$$

#7.

$$4x^2 - 4x + 5 \Rightarrow a(x-h)^2 + k$$

$$4 \left[x^2 - x + \left(\frac{1}{2}\right)^2 \right] + 5 - \left(\frac{1}{2}\right)^2 \cdot 4$$

$$4 \left[x - \frac{1}{2} \right]^2 + 5 - 1$$

$$= 4 \left[x - \frac{1}{2} \right]^2 + 4$$

#8 $f(x) = \sqrt{x-1}$ $D: x \geq 1, R: y \geq 0$
 $g(x) = x+2$ $D: x \in \mathbb{R}, R: y \in \mathbb{R}$

a) $f^{-1}(3) \times g^{-1}(3)$

$$f^{-1}(3) \Rightarrow (3)^2 = (\sqrt{y-1})^2$$

$$9 = y - 1 \Rightarrow y = 10$$

$$g^{-1}(3) \Rightarrow 3 = y + 2 \Rightarrow y = 1$$

$$\Rightarrow 10 \cdot 1 = \boxed{10}$$

b) $(f \circ g)^{-1}(3)$

$$f \circ g = \sqrt{x+2-1} = \sqrt{x+1}$$

$$(3)^2 = (\sqrt{y+1})^2 \Rightarrow 9 = y + 1$$

$$y = 8$$

#9 solve.

$$2 - \log_3(x+7) = \log_{\frac{1}{3}} 2x.$$

$$+ \log_3(x+7) = \log_{\frac{1}{3}} 2x$$

$$2 = \log_3(x+7) + \log_{\frac{1}{3}} 2x.$$

$$= \log_3(x+7) + \frac{\log_3 2x}{\log_3 \frac{1}{3}} = \log_3 3^{-1} = -1$$

$$2 = \log_3(x+7) - \log_3(2x)$$

$$2 = \log_3\left(\frac{(x+7)}{2x}\right)$$

$$\frac{x+7}{2x} = 3^2 = 9$$

$$\Rightarrow \frac{x+7}{2x} = 9 \Rightarrow \begin{aligned} x+7 &= 18x \\ 7 &= 17x \end{aligned}$$

$$x = \frac{7}{17}$$