IB Pre HL Addition and Subtraction Identities

$$\sin(x+y)=\sin x\cos y+\sin y\cos x$$

$$\sin(x-y)=\sin x\cos y-\sin y\cos x$$

$$\cos(x+y)=\cos x\cos y-\sin x\sin y$$

$$\cos(x-y)=\cos x\cos y+\sin x\sin y$$

$$\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x\tan y}$$

$$\tan(x-y)=\frac{\tan x-\tan y}{1+\tan x\tan y}$$

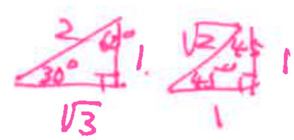
⇐ Given.

1. Find in simplest radical form: $\sin\frac{\pi}{12}$

$$\boxed{\frac{\pi}{12}=\frac{\pi}{3}-\frac{\pi}{4}}$$

$$\sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right)=\left(\sin\frac{\pi}{3}\right)\left(\cos\frac{\pi}{4}\right)-\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) \quad \left(=\frac{4\pi}{12}-\frac{3\pi}{12}=\frac{\pi}{12}\right)$$

$$=\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)-\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

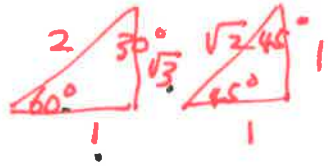


$$=\frac{\sqrt{3}}{2\sqrt{2}}-\frac{1}{2\sqrt{2}}=\checkmark\ \boxed{\frac{\sqrt{3}-1}{2\sqrt{2}}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$$

$$=\boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}\ \checkmark$$

2. Find in simplest radical form: $\tan 105^\circ$

$105^\circ = 60^\circ + 45^\circ$



$$\tan 105^\circ = \tan(60^\circ + 45^\circ)$$

$$= \frac{\tan 60^\circ + \tan 45^\circ}{1 - \tan 60^\circ \tan 45^\circ}$$

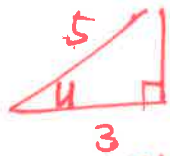
$$= \frac{\sqrt{3} + 1}{1 - \sqrt{3}\cdot 1} = \frac{(1+\sqrt{3})(1+\sqrt{3})}{(1-\sqrt{3})(1+\sqrt{3})} = \frac{1 + 2\sqrt{3} + 3}{1 - 3}$$

$$= \frac{4 + 2\sqrt{3}}{-2}$$

$$= -2 - \sqrt{3}$$

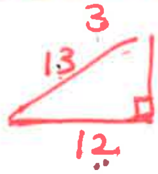
3. Find $\sin(u+v)$ if

$\sin u = \dfrac{4}{5}, \ 0 < u < \dfrac{\pi}{2}$ $\Rightarrow$



$4 \Rightarrow \cos u = \dfrac{3}{5}$

$\cos v = -\dfrac{12}{13}, \ \dfrac{\pi}{2} < v < \pi$ $\Rightarrow$



$5 \Leftarrow \sqrt{(13)^2 - (12)^2} = \sqrt{25} = 5$

$\sin v = \dfrac{5}{13}$

$\sin u \cdot \cos v + \sin v \cos u$

$$= \left(\frac{4}{5}\right)\left(-\frac{12}{13}\right) + \left(\frac{5}{13}\right)\left(\frac{3}{5}\right) = \frac{-33}{5\cdot 13} = \frac{-33}{65}$$

-48
15

② $\cos 2\theta = 2\cos^2\theta - 1$

$$= 2\left(\frac{2}{5}\right)^2 - 1 = \frac{8}{25} - \frac{25}{25} = \boxed{\frac{-17}{25}}$$

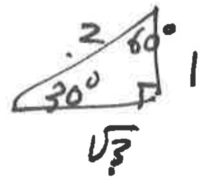
③ $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta} = \dfrac{2\left(\frac{-\sqrt{21}}{2}\right)}{1-\left(\frac{-\sqrt{21}}{2}\right)^2}$

$$= \frac{-\sqrt{21}}{\frac{4}{4}-\frac{21}{4}} = \frac{+\sqrt{21}}{\frac{+17}{4}} = \frac{4\sqrt{21}}{17}$$

4

(a) (i) Express $\cos\left(\frac{\pi}{6}+x\right)$ in the form $a\cos x - b\sin x$ where $a, b \in \mathbb{R}$.

(ii) Hence solve $\sqrt{3}\cos x - \sin x = 1$ for $0 \le x \le 2\pi$.

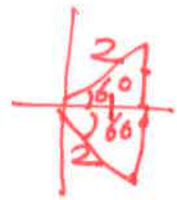


$\cos\frac{\pi}{6}\cdot\cos x - \sin\frac{\pi}{6}\cdot\sin x.$

$= \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x.$

$\cos\left(\frac{\pi}{6}+x\right) = \boxed{\frac{\sqrt{3}\cos x}{2} - \frac{\sin x}{2}} = \frac{1}{2}$

$\cos\left(\frac{\pi}{6}+x\right) = \frac{1}{2}.$



$\frac{\pi}{6} + x = \cos^{-1}\left(\frac{1}{2}\right) \Rightarrow$ ① $\frac{\pi}{6}+x = \frac{\pi}{3}$

$-\frac{\pi}{6} \quad -\frac{\pi}{6}$

$x = \frac{\pi}{6}$

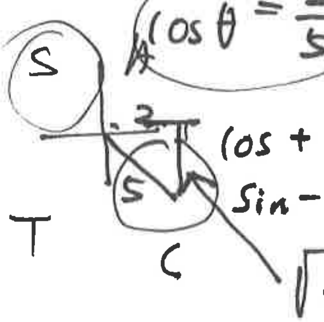
② $\frac{\pi}{6}+x = \frac{5\pi}{3}$

$x = \frac{10\pi}{6} - \frac{\pi}{6}$

$= \frac{9\pi}{6} = \boxed{\frac{3\pi}{2}}$

5 $\cos\theta = \frac{2}{5}$ and $\sin\theta < 0$, Find $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$

$\cos\theta = \frac{2}{5}$. $\sin\theta = \frac{-\sqrt{21}}{5}$



$\tan\theta = \frac{-\sqrt{21}}{2}$

$\sqrt{25-4} = \sqrt{21}$

① $\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{-\sqrt{21}}{5}\right)\left(\frac{2}{5}\right)$

$= \frac{-4\sqrt{21}}{25}$