$n \in \mathbb{Z}^+$ (1, 2, 3, 4 ...) ①

Pre HL : Ch 7 Arithmetic and Geometric Sequences

Arithmetic Sequences: $u_n = u_1 + (n-1)d$ and Geometric Sequences $u_n = u_1 r^{n-1}$

Warm UP:

1. List the first three terms of the specified sequence. Then tell whether the sequence is arithmetic, geometric, or neither.

a. $t_n = 5n + 2$ arithmetic.

$n=1$ 7 $\xrightarrow{+5}$ 12 $\xrightarrow{+5}$ 17

b. $t_n = \frac{n+1}{n+2}$

$n=1 : \frac{2}{3}$ $n=2 : \frac{3}{4}$ $n=3 : \frac{4}{5}$ neither

c. $t_n = 3^n$ Geometric.

d. $t_n = (-1)^n \cdot n$

$n=1$ -1 $n=3$ -3

$n=2$ 2 $n=4$ 4 neither

e. $t_n = 16 \cdot 2^{2n}$ Geometric.

f. $t_n = \sin(\frac{n\pi}{2})$

$n=1$ $\sin(\frac{\pi}{2}) = \sin(90°) = 1$

$n=2$ $\sin(\frac{2\pi}{2}) = \sin(\pi) = 0$

$n=3$ Neither.

2. Find a formula t_n.

a. 1, 4, 7, 10 .... A

$u_n = 1 + (n-1) \cdot 3 = -2 + 3n$

B. 0.3, 0.9, 2.7, 8.1..... G

$u_n = (0.3)(3)^{n-1}$

c. 1, 4, 9, 16, ...

$\Rightarrow 1^2, 2^2, 3^2, 4^2 ..$ N.

$u_n = n^2$

D. ½, 2/3, ¾, 4/5 ..... N

$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5} ..$

$u_n = \frac{n}{n+1}$.

$u_n = (0.3)(3)^{n} \cdot 3^{-1}$

$u_n = \frac{0.3}{3}(3)^n = 0.1 \cdot 3^n$

Geometric Sequences Example)

$k-1$, $2k$, *and* $21-k$ are consecutive terms of a geometric sequence. Find k.

$\times r$ $\times r$.

$\frac{k-1}{2k} = \frac{2k}{21-k}$. OR $\frac{2k}{k-1} = \frac{21-k}{2k}$.

$4k^2 = (k-1)(21-k)$

$4k^2 = -k^2 + 22k - 21$

$+k^2 - 22k + 21$ $\quad +k^2 - 22k + 21$

$5k^2 - 22k + 21 = 0$

$5k \quad -7 \quad -15$

$k \quad -3 \quad -7$

-22

$(5k-7)(k-3) = 0$

$k = 3$ OR $k = \frac{7}{5}$

(2)

3. a. How much money must be invested today so that in 2 years the amount $18,000 can be withdrawn from the account, if the money is invested at 8% (annual interest) compounded quarterly.

$$18000 = P\left(1+\frac{0.08}{4}\right)^{4\cdot 2}$$

$P = \$15,362.83$ OR $\$15,363$

b. If you have $8,000 to invest how much time would it take to reach the 18000?

$$18000 = 8000\left(1+\frac{0.08}{4}\right)^{4\cdot t}$$

$$\ln\frac{18}{8} = \ln\left(1+\frac{0.08}{4}\right)^{4t} \Rightarrow \ln 2.25 = (4t)\ln(1.02)$$

$$t = \left(\frac{\ln 2.25}{\ln 1.02}\right)\cdot\frac{1}{4} \approx 10.24 \text{ yrs.}$$

$\approx$ 11 yrs

4. The sum of three consecutive terms of a geometric sequence is 39 and their product is 729. Find these terms.

$a + ar + ar^2 = 39 \Rightarrow a(1+r+r^2) = 39$

$a\cdot(ar)\cdot(ar^2) = 729$

$\Rightarrow a^3\cdot r^3 = 729$

$(ar) = \sqrt[3]{729}$

$ar = 9$

$a = \frac{9}{r}$

$\Rightarrow \frac{9}{r}(1+r+r^2) = 39$

$\Rightarrow \left(\frac{9}{r} + 9 + 9r\right) = (39)\,r$

$9 + 9r + 9r^2 = 39r \rightarrow (3r-1)(r-3) = 0$

$9r^2 - 30r + 9 = 0$

$3r^2 - 10r + 3 = 0$

$r = \frac{1}{3}$, $a = 27$; $r = 3$, $a = 3$

(1) 27, 9, 3

OR

(2) 3, 9, 27

5. A geometric sequence $\{u_n\}$ has complex terms and is defined by $u_1 = 12i$ and $u_{n+1} = (2-i)\cdot u_n$ where $n \in \mathbb{Z}^+$. What are the first 3 terms of the sequence? What is an explicit formula for the sequence?

$u_1 = 12i$

$u_2 = (2-i)u_1 = (2-i)(12i) = 24i + 12$

$u_3 = (24i+12)(2-i) = 48i + 24 + 24 - 12i$

$= 48 - 36i$

Explicit Formula .. $u_n = (12i)(2-i)^{n-1}$