

IB Pre HL: Binomial Theorem

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Warm up:

Find the coefficient of x^3 in the expansion of $(2x - \frac{1}{x^2})^{11}$.

$${}^{11}C_r \cdot (2x)^{11-r} \cdot (-x^{-2})^r = Ax^3$$

$${}^{11}C_r \cdot 2^{11-r} \cdot (-1)^r \cdot (x^{11-r-2r}) = Ax^3$$

$${}^{11}C_r \cdot 2^{11-r} \cdot (-1)^r = A \quad | 11-4r=3 \quad r=2$$

$$\rightarrow {}^{11}C_2 \cdot 2^9 \cdot (-1)^2 = A$$

$$A = 55 \cdot 512$$

$$A = \boxed{28160}$$

More Examples

General term in the Binomial Expansion: $(a+b)^n = \sum_{r=0}^n \binom{n}{r} a^{n-r} b^r$

Example 1) The coefficient of x^4 in the expansion $(x+a)^6$ is 60. Find a .

$${}^6C_2 \cdot x^{6-2} \cdot a^2 = 60x^4$$

$$15a^2 = 60$$

$$a^2 = 4$$

$$a = \pm 2$$

Example 2) The coefficient of x^4 in the expansion $(1-2x)^n$ is 560. Find n .

$${}^nC_4 \cdot 1^{n-4} \cdot (-2x)^4 = 560x^4$$

$${}^nC_4 \cdot 16x^4 = 560x^4$$

$${}^nC_4 = 35$$

$$\boxed{n=7}$$

Example 3) Find K given that the constant term of $(kx + \frac{1}{\sqrt{x}})^9$ is -10^{-2} .

$${}^9C_r \cdot (kx)^{9-r} \cdot (x^{-\frac{1}{2}})^r = -\frac{10}{2} \cdot x^0$$

$${}^9C_r \cdot k^{9-r} \cdot x^{9-r-\frac{1}{2}r} = -\frac{10}{2}x^0$$

$${}^9C_r \cdot k^{9-r} = -\frac{10}{2} \quad | 9-r-\frac{1}{2}r=0 \quad r=6$$

$${}^9C_6 \cdot k^{9-6} = -\frac{10}{2}$$

$$84 \cdot k^3 = -\frac{10}{2}$$

$$k^3 = -\frac{1}{8}$$

$$\boxed{k = -\frac{1}{2}}$$

Practice)

1) Find the values of a and b if

$$(2-x)(1+\frac{1}{2}x)^6 = 2+ax+bx^2 \dots \dots \dots -\frac{1}{64}x^7$$

$$(ax = -x \binom{6}{0} \cdot 1^{6-0} \cdot (\frac{1}{2}x)^0) + 2 \binom{6}{1} \cdot 1^{6-1} \cdot (\frac{1}{2}x)^1$$

$$= -x \cdot 1 + 2 \cdot 3x \cdot 1x^2 = -x + 3x^2$$

$$ax = 5x$$

$$\boxed{a=5}$$

$$= -3x^2 + 2 \cdot \frac{15}{4}x^2$$

$$= -3x^2 + \frac{15}{2}x^2$$

$$bx^2 = \frac{9}{2}x^2$$

$$\boxed{b=\frac{9}{2}}$$

2) Find b given that the constant term of $(\sqrt{x} + \frac{b}{x})^9$ is -4032 .

$${}^9C_r \cdot (x^{\frac{1}{2}})^{9-r} \cdot (bx^{-1})^r = -4032$$

$${}^9C_r \cdot b^r \cdot x^{\frac{9}{2}-\frac{1}{2}r-r} = -4032$$

$${}^9C_r \cdot b^r = -4032 \quad | \frac{9}{2} - \frac{1}{2}r - r = 0$$

$$r=3$$

$${}^9C_3 \cdot b^3 = -4032$$

$$84b^3 = -4032$$

$$b^3 = -48$$

$$b = \sqrt[3]{-48}$$

$$b = -2\sqrt[3]{6}$$