

IB Pre HL: Binomial Theorem

Warm up:

Expand $(x+y)^6$

$$= x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$$

1	6	15	20	15	6	1
1	7	21	35	35	21	7
1	1	2	3	3	2	1
1	3	6	10	10	6	3
1	4	6	4	1		
1	1	1				

General term in the Binomial Expansion: $(a+b)^n = \sum_{r=0}^n \binom{n}{r} a^{n-r} b^r$

Example 1) What is the coefficient of x^6 in $(x^2 + \frac{4}{x})^{12}$?

$${}_{12}C_r (x^2)^{12-r} (4x^{-1})^r = Ax^6$$

$${}_{12}C_r \cdot 4^r (x^{24-2r} \cdot x^{-r}) = Ax^6$$

$${}_{12}C_r \cdot 4^r (x^{24-3r}) = Ax^6$$

$$24 - 3r = 6$$

$$r = 6$$

$${}_{12}C_6 \cdot 4^6 = 924 \cdot 4^6 = 924 \cdot 4096 = 3784704$$

Example 2) In the expansion of $(x^2 - \frac{4}{x})^{12}$ find a) the coefficient of x^6 and b) the constant term.

a) ${}_{12}C_r (x^2)^{12-r} (4x^{-1})^r = Ax^0$

$${}_{12}C_r \cdot 4^r (x^{24-2r} \cdot x^{-r}) = Ax^0$$

$$24 - 3r = 0$$

$$r = 8$$

$${}_{12}C_8 \cdot 4^8 = 32449320$$

Example 3) Find the coefficient of x^5 in the expansion of $(x+2)(x^2+1)^8$

$x^2 + 8$

$${}_{8}C_r (x^2)^{8-r} \cdot 1^r = Ax^4$$

$${}_{8}C_r \cdot x^{16-2r} = Ax^4$$

$$16 - 2r = 4$$

$$A = 8C_6 = 28$$

$$3x [{}_{8}C_2 (2x)^{6-2} (1)^2] + A [{}_{8}C_4 (2x)^{4-4} (1)^4]$$

$$= 3x \cdot 280^2 x^4 + 192A x^5$$

$$= (7280^2 + 192A) x^5$$

Practice)

1

(a) Find the term in x^3 in the expansion of $(3x+4)(2x+B)^6$.
 Mina and Norbert each have a fair cubical die with faces labelled 1, 2, 3, 4, 5 and 6; they throw it to decide if they are going to eat a cookie.

Mina throws her die just once and she eats a cookie if she throws a four, a five or a six. Norbert throws his die six times and each time eats a cookie if he throws a five or a six.

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(b) Calculate the probability that five cookies are eaten.

Find the constant term in the expansion of $(x - \frac{2}{x})^3$

$$(x^4 - 4x^3 \cdot 2x^{-4} + 6x^2 \cdot 4x^{-2} - 4x \cdot 8x^{-2} + 16x^{-4}) \cdot (x^6 + 3x^4 \cdot 2x^{-4} + 3x^2 \cdot 4x^{-2} + 8x^{-3})$$

$$= (x^4 - 8x^2 + 24 - 32x^{-2} + 16x^{-4})(x^6 + 6x^3 + 12 + 8x^{-3})$$

$$A = 288$$

$$= \frac{72}{1458} = \frac{4}{81}$$