

Calculators is allowed!! Show all your work!

1. Find the exact value (s) in simplest form.

a.  $\tan\left(\frac{5\pi}{12}\right) = \tan\left(\frac{2\pi}{12} + \frac{3\pi}{12}\right)$   
 $= \tan\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$

b.  $\sin(105^\circ) = \sin(60 + 45^\circ)$

$= \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ$   
 $= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$   
 $= \frac{1 + \sqrt{3}}{\sqrt{3} + 1} \cdot \sqrt{3} = \frac{1 + \sqrt{3}}{3 - 1} = \frac{4 + 2\sqrt{3}}{2} = 2 + \sqrt{3}$



$\sqrt{3} = \frac{\tan\left(\frac{\pi}{6}\right) + \tan\left(\frac{\pi}{4}\right)}{1 - \tan\left(\frac{\pi}{6}\right)\tan\left(\frac{\pi}{4}\right)} = \frac{\left(\frac{1}{\sqrt{3}} + 1\right) \cdot \sqrt{3}}{1 - \frac{1}{\sqrt{3}}}$

2. Given  $\tan(2x) = \frac{5}{12}$ , find the possible values of  $\tan(x)$ .

$\frac{5}{12} = \frac{2 \tan x}{1 - \tan^2 x} \Rightarrow 5 - 5 \tan^2 x = 24 \tan x$

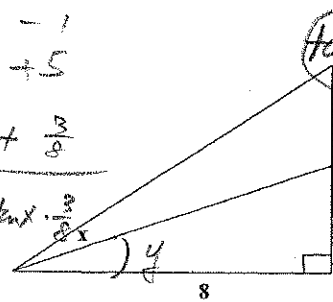
$5 \tan^2 x + 24 \tan x - 5 = 0 \Rightarrow (5 \tan x - 1)(\tan x + 5) = 0$

3. Find the exact value of  $\tan x$ .

$\frac{5}{1} = \frac{-1}{+5}$

$\tan x = \frac{1}{5}$        $\tan x = -5$

$\tan(x+y) = \frac{7}{8} = \frac{\tan x + \tan y}{1 - \tan x \cdot \tan y} = \frac{\tan x + \frac{3}{8}}{1 - \tan x \cdot \frac{3}{8}}$



$\Rightarrow 7(1 - \tan x \cdot \frac{3}{8}) = 8(\tan x + \frac{3}{8}) \Rightarrow 7 - \frac{21}{8} \tan x = 8 \tan x + 3$

6. Solve for  $0 \leq \theta \leq 2\pi$ .

a.  $\sqrt{2} \sin \theta = \tan \theta$

$\Rightarrow 4 = \frac{85}{8} \tan x \xrightarrow{\frac{64}{85}} \tan x = \frac{32}{85}$

$\sqrt{2} \sin \theta = \frac{\sin x}{\cos x}$

$\sin \theta = 0 \Rightarrow x = 0, 2\pi$

$\sqrt{2} \sin \theta - \sin x \cdot \sec x = 0 \Rightarrow \sin \theta [\sqrt{2} - \sec x] = 0$

$\sec \theta = \sqrt{2}$

$\Rightarrow \frac{\pi}{4}, \frac{7\pi}{4}$

b.  $\cos 2\theta + \sin \theta = 0$

$1 - 2 \sin^2 \theta + \sin \theta = 0$

$\sin \theta = \frac{-1}{2} \Rightarrow \frac{7\pi}{6}, \frac{11\pi}{6}$

$2 \sin^2 \theta - \sin \theta - 1 = 0$

$(2 \sin \theta + 1)(\sin \theta - 1) = 0 \Rightarrow \sin \theta = 1 \Rightarrow 0, 2\pi$

7. Find a and b for  $y = \cos 2\theta + \sqrt{3} \sin 2\theta = a \cos[2(\theta - b)]$ .

$\frac{2}{3} \sqrt{3} \sin 2\theta + \cos 2\theta = a \cos[2(\theta - b)]$   
 $2\theta = \frac{\pi}{3} \quad \theta = \frac{\pi}{6} \Rightarrow 2 \cos[2(\theta - \frac{\pi}{6})] = y$