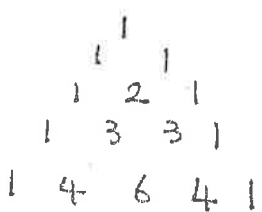


1. Expand  $(x - \frac{1}{x^2})^3 (2-x)^4$  and find the constant term.

$$= [x^3 + 3(x)^2(-x^{-2}) + 3(x)(-x^{-2})^2 + (-x^{-2})^3] \times [2^4 + 4(2)^3(-x) + 6(2)^2(-x)^2 + 4(2)(-x)^3 + (-x)^4]$$

$$= [x^3 - 3x + 3x^{-2} - x^{-4}] \times [16 - 32x + 24x^2 - 8x^3 + x^4]$$

$$= (3)(2^4) + (3)(-8) = \boxed{-72} \text{ is the constant term}$$



2. Find the value of k when the coefficient of  $x^4$  is 1125 in the expansion of  $(x - \frac{k}{x^2})^{10}$ .

$${}^{10}C_r (x)^{10-r} (-k/x^2)^r = 1125 x^4$$

$$10-r-2r=4$$

$$6=3r \quad \boxed{r=2}$$

$$\Rightarrow {}^{10}C_2 (-k)^2 = 1125$$

$$k^2 = 25$$

$$\boxed{k = \pm 5}$$

3.

Simplify:  $\frac{(n+1)! - (n-1)!}{n!}$

$$= \frac{(n-1)!(n)(n+1) - (n-1)!}{n(n-1)!} = \frac{n(n+1) - 1}{n} = \boxed{\frac{n^2 + n - 1}{n}}$$

More practice :

$$\frac{(n+2)! - n!}{(n+1)!} = \frac{n!(n+1)(n+2) - n!}{n!(n+1)}$$

$$= \frac{n^2 + 3n + 2 - 1}{n+1} = \boxed{\frac{n^2 + 3n + 1}{n+1}}$$

4. What is the coefficient of  $x^6$  in  $(x^2 - \frac{3}{x})^{12}$ ?

$$a = x^2$$

$$b = -3x^{-1}$$

$$n = 12$$

$$= {}^{12}C_r (x^2)^{12-r} (-3x^{-1})^r = C \cdot x^6$$


$$2(12-r) - r = 6$$


$$24 - 3r = 6 \quad 3r = 18 \quad r = 6$$

$$\Rightarrow {}^{12}C_6 (-3)^6 = \boxed{673596}$$

Log

5. 48 people are about to get on a double-decker bus which seats 24 people on each level. However, 8 people refuse to travel upstairs, and 6 people refuse to travel downstairs. How many ways are there of choosing which passengers travel upstairs and which passengers travel downstairs?

24   $\Rightarrow (-6)$  18 seats  $\Rightarrow$  Total #s of people to sit anywhere  
34 people

24   $\Rightarrow (-8)$  16 seats  ${}_{34}C_{18} = 2,203,961,430$

6. 10 people came to a book club to discuss "You are joking, Mr. Feynman". The seats are arranged in a circle facing toward center.

a. How many different ways can the seats be arranged ?

$$\frac{10!}{10} = 362,880$$

b. How many different ways can the seats be arranged when the couple, John and Jill, in the group are allowed to sit together?

$$2 \times 8! = 80,640$$