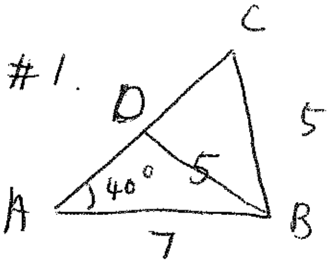
#1.



SSA ⇒ Ambiguous Case .

$\frac{\sin 40^\circ}{5} = \frac{\sin C}{7}$ $\quad C = \sin^{-1}\left(\frac{7\sin 40^\circ}{5}\right)$

$C \approx 64.1^\circ$

$A\hat{D}B = C' \approx 115.9^\circ$

$C\hat{D}B = 64.1^\circ$

$D\hat{B}C = 180 - (64.1)\cdot 2 = 51.8$.

$CD = \sqrt{5^2 + 5^2 - 2\cdot 5\cdot 5\cdot \cos(51.8)^\circ} \approx \boxed{4.37 \text{ units}}$

2. $A_{\text{shaded}} = \left(\frac{1}{2}\right)(\theta')(2)^2 = 3\pi$. where $\theta' = (2\pi - \theta)$ radians.

(a) $= 2\theta' = 3\pi$

$\theta' = \frac{3\pi}{2}$ radians $\qquad \theta = \frac{4\pi}{2} - \frac{3\pi}{2} = \frac{\pi}{2}$ radians.

Circumference of the whole Circle

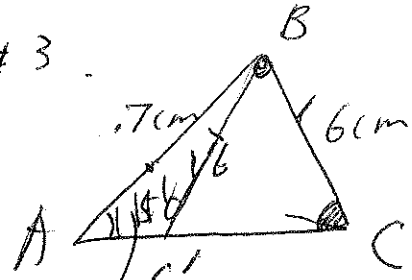
(b) $2\pi : (\pi)(4)$

$\frac{3\pi}{2} : X$ ← Arc length of the logo

$X = \frac{(\cancel{4}^{2}\cancel{\pi})\left(\frac{3\pi}{2}\right)}{\cancel{2}\cancel{\pi}} = 3\pi$ cm .

perimeter of loge : $\boxed{3\pi + 4 \text{ cm}}$

#3.



$\Rightarrow$ Ambiguous case

$$C = \sin^{-1}\left[\frac{7\sin 50^\circ}{6}\right] \approx 63.3^\circ$$

$$\frac{\sin 50^\circ}{6} = \frac{\sin C}{7}$$

$$B \approx (180^\circ - 63.3^\circ - 50) \approx 66.7^\circ$$

$$\angle BC'A = 180^\circ - 63.3 \approx 116.7^\circ$$

$$\angle ABC' = 180 - 116.7 - 50$$

$$\approx 13.3^\circ$$

$$\triangle ABC = \frac{1}{2}(7)(6)(\sin 66.7^\circ)$$

$$\approx 19.3$$

$$\triangle AB'C = \left(\frac{1}{2}\right)(7)(6)(\sin 13.3^\circ)$$

$$\approx 4.83$$

$\Leftarrow$ Less than 10 cm².

#4. ① If $n=1$ $\Rightarrow$ $7^{8+3}+2 = 1977326745$ is divisible by 5

True for $n=1$

② If $n=k$ $k \in \mathbb{Z}^+$,

Assume $(7^{8k+3}+2) = 5A$. where A is a positive integer

$7^{8k+3} = 5A - 2$

③ If $n=k+1$ $\Rightarrow$ $7^{8(k+1)+3}+2$

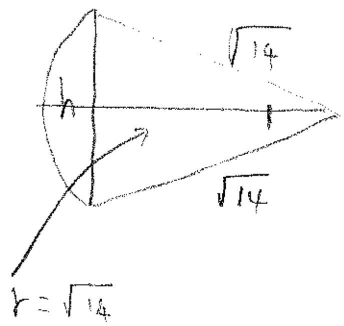
$= \boxed{7^{8k+3}}^{+8} + 2$

$= 7^8(5A-2)+2 = 5 \cdot A \cdot 7^8 - 11529600$

$= 5[7^8 A - 2305920]$ is divisible by 5.

④ $\therefore$ $7^{8n+3}+2$ is divisible by 5 for $n \in \mathbb{N}$

#5. $A=\left(\frac{1}{2}\right)(1)(r)^2=7$ $\qquad r=\sqrt{14}$

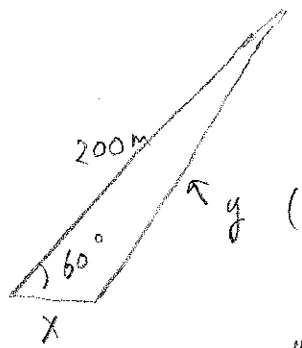


$h=\sqrt{14+14-2\sqrt{14}\sqrt{14}\cdot\cos(1)}$ ⇐ radian mode.

≈ 3.59 cm

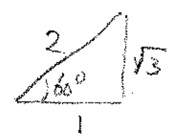
$\sqrt{14}$ cm by 3.59 cm.

#6.



y (Cable along the sea)

$\cos 60° = \frac{\sqrt{3}}{2}$



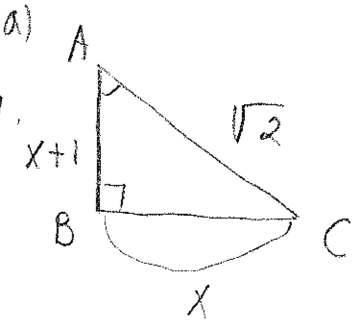
$y=\sqrt{(200)^2+(x)^2-2x\cdot(200)\cos 60°}$

$=\sqrt{40000+x^2-\cancel{2}\cdot x(200)\frac{1}{\cancel{2}}}$

$\boxed{\text{Total cable (cost)} = 80\sqrt{40000+x^2-200x}+20\cdot x.}$

a)

#7.



$\cos A = \frac{x+1}{\sqrt{2}}$ $\Rightarrow \cos A - \sin A$

$\sin A = \frac{x}{\sqrt{2}}$ $= \frac{x+1}{\sqrt{2}} - \frac{x}{\sqrt{2}} = \boxed{\frac{1}{\sqrt{2}}}$

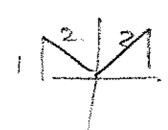
b)

$(\cos A - \sin A)^2 = \left(\frac{1}{\sqrt{2}}\right)^2$

$\Rightarrow \cos^2 A + \sin^2 A - 2\cos A\sin A = \frac{1}{2}$ $\quad (2\cos A \sin A = \sin 2A)$

$\Rightarrow 1 - 2\cos A\sin A$

$0 \le A \le 180°$

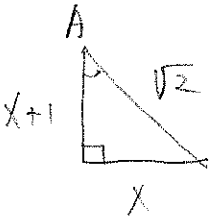


$\Rightarrow 1 - \sin 2A = \frac{1}{2} \Rightarrow \sin 2A = \frac{1}{2}$

$\boxed{A=15° \text{ OR } A=75°} \leftarrow 2A = 30° \text{ OR } 150°$

#7. $\sin A = \frac{\wedge}{\sqrt{2}} = \frac{x\sqrt{2}}{2}$.

C. $(x+1)^2 + x = 2$



$x^2 + 2x + 1 + x^2 = 2$ $\qquad 4+8$

$2x^2 + 2x - 1 = 0$

$$x = \frac{-2 \pm \sqrt{(2)^2 + (4)(2)(1)}}{2\cdot 2} = \frac{-2 \pm \sqrt{12}}{4}$$

$$= \frac{-2 \pm 2\sqrt{3}}{4} = \frac{-1 \pm \sqrt{3}}{2} \Leftarrow$$ Cannot have negative

$$\sin A = \frac{\frac{\sqrt{2}(\sqrt{3}-1)}{2\sqrt{2}}}{2} = \frac{\sqrt{3}-1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$$

#8. 1) When $n=1 \Rightarrow 1 = \left(\frac{1}{3}\right)(1)(4-1) = 1$

The Formula Works for $n=1$.

Formula:

$$\sum_{i=1}^{n} (2i-1)^2 = \frac{1}{3} n (4n^2 - 1)$$

for $n \in \mathbb{Z}^+$

2) If $n = k$ where $k \in \mathbb{Z}^+$

Assume $\sum_{i=1}^{k} (2i-1)^2 = \frac{1}{3} k (4k^2 - 1)$

$= \frac{1}{3} k (2k-1)(2k+1)$

3) If $n = k+1 \Rightarrow \sum_{i=1}^{k} (2i-1)^2 + [2(k+1) - 1]^2$

$= \frac{1}{3} k (4k^2 - 1) + [2k+2-1]^2$

$= \frac{1}{3} k (4k^2 - 1) + [2k+1]^2$

$= \frac{1}{3} k (2k+1)(2k-1) + (2k+1)^2$

$= \frac{1}{3} (2k+1) [k(2k-1) + 3(2k+1)]$

$= \frac{1}{3} (2k+1) [2k^2 - k + 6k + 3] = \frac{1}{3} (2k+1)(2k^2 + 5k + 3)$

$= \frac{1}{3} (2k+1)(2k+3)(k+1)$

$\boxed{\frac{1}{3} (k+1)(2(k+1)-1)(2(k+1)+1)}$

$\therefore$ The formula always works for $n \in \mathbb{Z}^+$.

#9. $9^{n+2}-4^n$ is divisible by 5 for $n \in \mathbb{Z}^+$.

1) If $n=1$ $9^3-4^1=725$ is divisible by 5.

2) If $n=k$, Assume $9^{k+2}-4^k$ is divisible by 5.

$$9^{k+2}-4^k = 5\cdot A$$ where A is a positive constant.

$$9^{k+2} = 5\cdot A + 4^k$$

3) If $n=k+1$, $9^{(k+1)+2}-4^{k+1}$

$$\left(9^{k+2}\right)\cdot 9 - 4^{k+1}$$

$$=\left(5\cdot A+4^k\right)\cdot 9-4^k\cdot 4$$

$$=45A+9\cdot 4^k-4\cdot 4^k = 45A+5\cdot 4^k$$

$$=5(9A+4^k)$$

which is divisible by 5.

4) $\therefore$ $9^{n+2}-4^n$ is always divisible by 5 for $n \in \mathbb{Z}^+$