

#1.

$$z = (2-i)(z+2)$$

$$z = 2z + 4 - zi - 2i$$

$$z + zi - 2z = 4 - 2i$$

$$-z + zi = 4 - 2i$$

$$z(-1+i) = 4-2i$$

$$z = \frac{(4-2i)(-1-i)}{(-1+i)(-1-i)} = \frac{-4-4i+2i+2(i)^2}{1+1}$$

$$= \frac{-6-3i}{2} = \boxed{-3-i}$$

#2

$$\left. \begin{aligned} 2(a+bi) &= (a-bi) + 6-2i \\ \boxed{2a} + \boxed{2bi} &= \boxed{(a+6)} + \boxed{(-b-2)i} \end{aligned} \right) \Rightarrow$$

$$2a = a+6 \Rightarrow a=6$$

$$2b = -b-2$$

$$3b = -2$$

$$\boxed{\begin{aligned} b &= -\frac{2}{3} \\ a &= \frac{2}{3} \end{aligned}}$$

$$\Rightarrow \boxed{z = \frac{2}{3} + \frac{2}{3}i}$$

#3.

$$\frac{i}{1} = \frac{z+i}{z+2}$$

$$\boxed{z+i} = \boxed{zi} + \boxed{2i}$$

$$z - zi = 2i - i$$

$$z(1-i) = i$$

$$z = \frac{(i)(1+i)}{(1-i)(1+i)} = \frac{i-1}{2} = \boxed{-\frac{1}{2} + \frac{1}{2}i}$$

#4. $2+i \Rightarrow \boxed{2-i} = x.$

Sum of ^{the} roots: 4

Product of the roots: $(2+i)(2-i)$
 $= 4+1 = 5$

\Rightarrow Quadratic Equation: $x^2 - 4x + 5$

$$\begin{array}{r}
 x^2 - 4x + 5 \quad \overline{) \quad x^3 - 6x^2 + 13x - 10} \\
 \underline{- \quad x^3 - 4x^2 + 5x} \\
 -2x^2 + 8x - 10 \\
 \underline{- \quad -2x^2 + 8x - 10} \\
 0
 \end{array}$$

$\boxed{x=2}$

The roots.

$\boxed{2+i, 2-i, \cancel{2}}$

#5 $\frac{2+bi}{1-bi} \neq \frac{7+9i}{10}$

$\Rightarrow 20 + 10bi = \overline{(7+9i)(1-bi)}$

$$\begin{aligned}
 \Rightarrow 20 + 10bi &= \overline{-7 + 7bi - 9i + 9b(i)^2} \\
 &= \overline{-(7-9b) + i(-9+7b)}
 \end{aligned}$$

(confirm.)

$27 = -9b \quad \boxed{b = -3}$

$10b = -9 + 7b$

$3b = -9 \quad \boxed{b = -3}$

#6. $z_1 = \frac{a}{1+i}$ $z_2 = \frac{b}{1-2i}$

$z_1 = \frac{a(1+i)}{2}$ $z_2 = \frac{b(1-2i)}{5}$

$\left(\frac{a}{2} + \frac{a}{2}i\right) + \left(\frac{b}{5} - \frac{2b}{5}i\right) = 3$

$\frac{a}{2} + \frac{b}{5} = 3$ $\frac{a}{2} - \frac{2b}{5} = 0$

$\frac{5a + 2b}{10} = 3$

$5a + 2b = 30$

$5a - 4b = 0$

$- \quad 5a + 2b = 30$

\Rightarrow solve the system of Equation.

$5a + 10 = 30$

$-6b = -30$

$5a = 20$ $a = 4$

$b = 5$ $a = 4$

#6. Sum: $(a+ai) + (a-ai) = 2a$
of the roots.

$x_1: a+ai$

$x_2: a-ai$

Product
of the roots: $(a+ai)(a-ai)$
 $= a^2 + a^2 = 2a^2$

$\Rightarrow 2a = 6 \Rightarrow a = 3$

$\Rightarrow 2a^2 = b \Rightarrow b = (2)(9) = 18$