

#1. $a(x - (2 - 3i))(x - (2 + 3i))(x - 2)(x - 1) = f(x)$ [Quartic Equation]

$\Rightarrow (x - 2 + 3i)(x - 2 - 3i) = (x - 2)^2 - (3i)^2 = x^2 - 4x + 4 + 9 = x^2 - 4x + 13$

$\Rightarrow (x - 2)(x - 1) = x^2 - 3x + 2$

$\Rightarrow a[(x^2 - 4x + 13)(x^2 - 3x + 2)] = f(x) \in (0, 5)$

$5 = a[13 \cdot 2] \Rightarrow a = \frac{5}{26}$

14
13

$f(x) = \frac{5}{26} [x^4 - 3x^3 + 2x^2 - 4x^3 + 12x^2 - 8x + 13x^2 - 39x + 26]$

$= \frac{5}{26} [x^4 - 7x^3 + 27x^2 - 47x + 26]$

$= \left[\frac{5}{26}x^4 - \frac{35}{26}x^3 + \frac{135}{26}x^2 - \frac{235}{26}x + 5 \right]$

#2. $\frac{p: B \Rightarrow \pm 1, \pm 2, \pm 4, \pm 8}{q: 2 \Rightarrow \pm 1, \pm 2} \Rightarrow \left\{ \frac{p}{q} \in \{ \pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{2} \} \right\}$

#3. $f(x) = k(x + 1)(x - 2)^2(x - 3) \in (0, -4)$

$-4 = k(1)(-2)^2(-3) \Rightarrow k = \frac{-4}{-12} = \frac{1}{3}$

$f(x) = \frac{1}{3}(x + 1)(x - 2)^2(x - 3)$

#4. $p(x) = (x^2 + 1)(x^2 + x - 1) + ax + b$

$p(2) = -6 \Rightarrow -6 = (2^2 + 1)(2^2 + 2 - 1) + 2a + b$

$\Rightarrow -6 = 5(5) + 2a + b$

$\Rightarrow 2a + b = -31$

$p(-1) = 5 \Rightarrow 5 = (1 + 1)(1 - 1 - 1) - a + b$

$5 = -2 - a + b$

$\Rightarrow -a + b = 7$

$p(x) = (x^4 + x^3 - \frac{35}{3}x - \frac{20}{3})$

$b = 7 + a$
 $= 7 - \frac{38}{3}$

Solve system

$3a = -38$

$a = \frac{-38}{3}$

$b = \frac{-21}{3}$

$= \frac{-21}{3} - \frac{38}{3}$