Chapter 1

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Quadratic Formula: $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

Discriminant: b^2-4ac

Optimization: Finding the vertex of a function

If $a>0$, the minimum value of y: $\frac{-b}{2a} = x$

If $a<0$, the maximum value of y is $x = \frac{-b}{2a}$

Sum & product: $y = ax^2 + bx + c$

sum: $\frac{-b}{a}$ product: $\frac{c}{a}$

A rectangle has length 3 cm longer than the width. Its area is 42 cm². Find its width.

$x(x+3) = 42\,cm^2$ Translate words into algebra

$x^2+3x = 42$

$x^2+3x-42=0$

$x = \dfrac{-3 \pm \sqrt{9-4(-42)}}{2}$

$x = \dfrac{-3 \pm \sqrt{177}}{2}$

$x = -3 \pm 13.30$

Solve the equation

$\cancel{x = -8.15}$ $x = 5.15$

Choose the acceptable value(s)

The width is 5.15 cm Answer in a sentence

$x^2+4x+1 = 0$

$x^2+4x+4-4+1$

$\boxed{(x+2)^2-3}$

Chapter 2

Function Notation = $f(x)$

Function V.S. Relation : Function is a relation that doesn't have more than one y-value for every x-value.

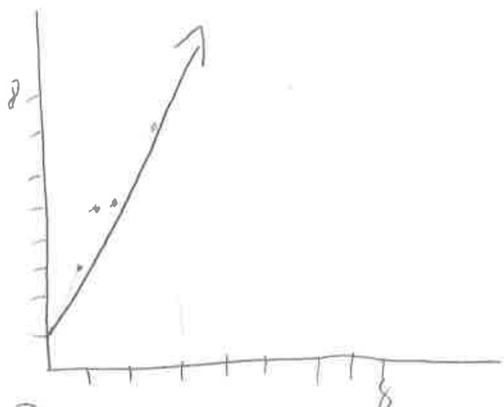
Piecewise - Defined Functions : The modulus of x, $|x| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$

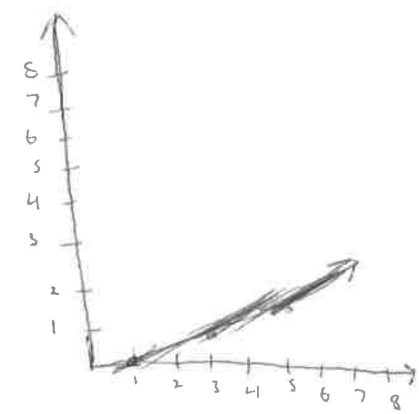
Composite Function : Composite Function of $f(x)$ and $g(x)$ will convert x into $f(g(x))$. Represented by $f \circ g$. $(f \circ g)(x) = f(g(x))$ or $f \circ g : x \to f(g(x))$

$$f(x) = 2x + 1$$

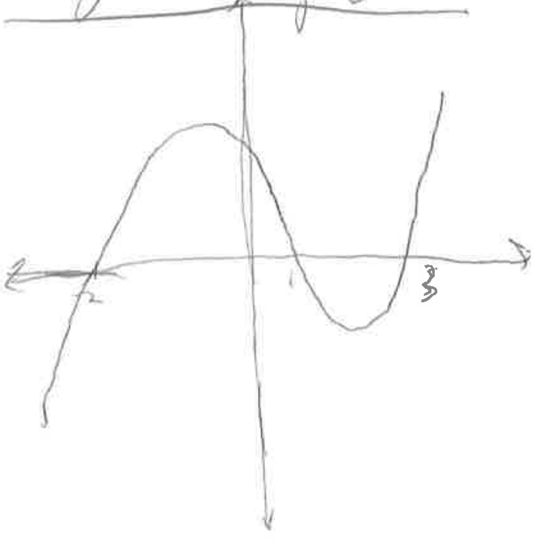
$$f^{-1}(x)$$

(x and y points swapped)





Sign Diagram



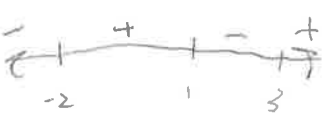
$$\frac{3x+2}{1-x} > 4$$

$3x+2 > 4(1-x)$

$3x+2 > 4-4x$

$7x > 2$

$x > \frac{2}{7}$



UNIT 3 & 4 → EXPONENTS and LOGARITHMS

Ben

$$x^{y}$$

y ← exponent

x ← base

exponent laws

* $a^b \cdot a^c = \boxed{a^{b+c}}$
* $\dfrac{a^b}{a^c} = \boxed{a^{b-c}}$
* $(a^b)^c = \boxed{a^{b\times c}}$
* $(ab)^c = \boxed{a^c b^c}$
* $\left(\dfrac{a}{b}\right)^c = \boxed{\dfrac{a^c}{b^c}}$ (b cannot be zero)
* $a^0 = \underline{1}$ (a cannot be zero)
* $a^{-b} = \boxed{\dfrac{1}{a^b}}$ / $\dfrac{1}{a^{-b}} = \boxed{a^b}$ ← a cannot be zero
* $a^{\frac{1}{b}} = \boxed{\sqrt[b]{a}}$ / $a^{\frac{b}{c}} = \boxed{\sqrt[c]{a^b}}$

algebraic expansion & factorization

$a(b+c) = ab + ac$

$(a+b)(c+d) = ac + ad + bc + bd$

$(a+b)(a-b) = a^2 - b^2$

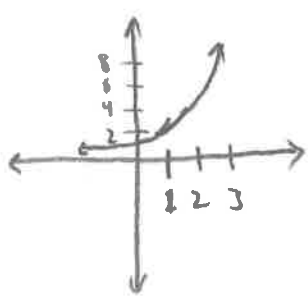
$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

exponential function

$y = 2^x$

x	y
-3	$\frac{1}{8}$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8



Exponent rules

$x^2 \cdot x^3 = x^5$ => add exponents
$x^m \cdot x^n = x^{m+n}$ when multiplying

$x^4 \div x^2 = x^2$ => subtract exponents
$x^m \div x^n = x^{m-n}$ when dividing

$(x^2)^3 = x^6$ => multiply exponents
$(x^m)^n = x^{mn}$

$(xy)^m = x^m y^m$

$x^0 = 1, x \neq 0$

$x^{-n} = \frac{1}{x^n}$

$\frac{1}{x^{-n}} = x^n, x \neq 0$

Algebraic Expansion

$a(b+c) = ab + ac$

$(a+b)(c+d) = ac + ad + bc + bd$

$(a+b)(a-b) = a^2 - b^2$

$(a+b)^2 = a^2 + 2ab + b^2$

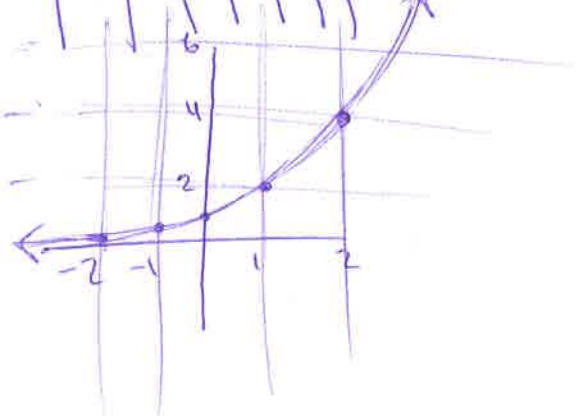
$(a-b)^2 = a^2 - 2ab + b^2$

Graphing

$y = b^x$

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

$y = 2^x$



Factorization

① $2^{n+3} + 2^n$

$2^n 2^3 + 2^n$

$2^n(2^3+1)$

$2^n \cdot 9$

② $\frac{3^n + 6^n}{3^n}$

$\frac{3^n + 2^n 3^n}{3^n}$

$\frac{\cancel{3^n}(1+2^n)}{\cancel{3^n}}$

$1 + 2^n$

The Natural Exponent e

$e \approx 2.7183$

"irrational, like pi

$\log_e = \ln$

ln can be used like log

Change of Base:

$$\log_x Y = \frac{\log Y}{\log x}$$

Example: $\log_5 4 = \frac{\log 4}{\log 5}$

Laws of Logarithms

$a = 10^{\log a}$ for any $a > 0$

$\log 10^x = x$

Doesn't have to be base 10:

$$\boxed{\begin{array}{l} x = \log_a a^x \\ x = a^{\log_a x} \quad \text{for } x > 0 \end{array}}$$

In any base c where $c \neq 1$, $c > 0$:

If A and B are both positive then:

$$\log_c A + \log_c B = \log_c (AB)$$

$$\log_c A - \log_c B = \log_c \left(\frac{A}{B}\right)$$

$$n \log_c A = \log_c A^n$$

Solving Equations:

$e^x = 30$

$\ln e^x = \ln 30$

$x = \ln 30$

$e^x + 2 = 3e^{-x}$

$e^x(e^x + 2) = e^x(3e^{-x})$

$e^{2x} + 2e^x - 3 = 0$

$(e^x + 3)(e^x - 1) = 0$

$\cancel{e^x = -3}$

$e^x = 1 \quad \ln e^x = \ln 1$

$\cancel{x} = \ln 1$

$e^{2x} = 2e^x$

$e^{2x} - 2e^x = 0$

$e^x(e^x - 2) = 0$

$\cancel{e^x = 0}$

$e^x = 2$

$\ln e^x = \ln 2$

$x = \ln 2$

UNIT 3 + 4 → exponents + logarithms

exponent rules

$a^b \cdot a^c = \boxed{a^{b+c}}$ $\quad (a^b)^c = \boxed{a^{bc}}$ $\quad \left(\frac{a}{b}\right)^c = \boxed{\frac{a^c}{b^c}}$ $\quad a^{-b} = \boxed{\frac{1}{a^b}}$

* "b" cannot be 0

$\frac{a^b}{a^c} = \boxed{a^{b-c}}$ $\quad (ab)^c = \boxed{a^c b^c}$ $\quad a^0 = \boxed{1}$ $\quad \frac{1}{a^{-b}} = \boxed{a^b}$

* "a" cannot be 0

algebraic expansion + factorisation

$a(b+c) = \boxed{ab + ac}$ $\quad (a+b)(a-b) = \boxed{a^2 - b^2}$

$(a+b)(c+d) = \boxed{ac + ad + bc + bd}$ $\quad (a+b)^2 = \boxed{a^2 + 2ab + b^2}$

$(a-b)^2 = \boxed{a^2 - 2ab + b^2}$

example: simplify $x^{-\frac{1}{2}}(x^{\frac{3}{2}} + 2x^{\frac{1}{2}} - 3x^{-\frac{1}{2}})$

$= x^{-\frac{1}{2}}(x^{\frac{3}{2}}) + x^{-\frac{1}{2}}(2x^{\frac{1}{2}}) - x^{-\frac{1}{2}}(3x^{-\frac{1}{2}})$

$= x^1 + 2x^0 + 3x^{-1}$

$= \boxed{x + 2 + \frac{3}{x}}$

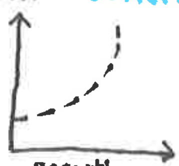
example: factorise $4^x - 9$

$= (2^x)^2 - 3^2$

$= \boxed{(2^x + 3)(2^x - 3)}$

growth + decay

growth: when populations of animals / bacteria increase exponentially.



growth

decay: when radioactive substances depreciate exponentially.

decay

Chapter 6: Polynomials

- $ax^2+bx+c=0$ where $a \neq 0$ and $a,b,c \in \mathbb{R}$
- Find the roots with quadratic formula:
  ↳ $x = \frac{-b \pm \sqrt{\Delta}}{2a}$ $\Delta = b^2 - 4(a)(c)$
  ↳ When: $\Delta > 0 \rightarrow$ 2 real solutions
  $\Delta = 0 \rightarrow$ 1 solution (real)
  $\Delta < 0 \rightarrow$ 2 imaginary solutions
- Imaginary solutions are complex numbers $(a+bi)$
  ↳ a & b are real, i is $\sqrt{-1}$
  ↳ if a is 0 it is purely imaginary, when b is 0 it's real
  ↳ written in form $z = a+bi$, with a conjugate of $z^* = a-bi$
- When a quadratic equation with real roots has a complex root, it's conjugate is also a real root
  ↳ Ex: x^2-2x+5. $x = 1+2i$, $x = 1-2i$
- Polynomials can be up to many degrees, and dividing them is like normal division
- Factor theorem, if 2 is a zero of $P(x)$, then $(x-2)$ is a factor
- Sum and product, for $ax^2+bx+c=0$, $a \neq 0$, sum: $\frac{-b}{a}$, prod: $\frac{c}{a}$
  (if the degree is odd, the product becomes $\frac{-c}{a}$
- When graphing, degree and coefficient determine end behavior.

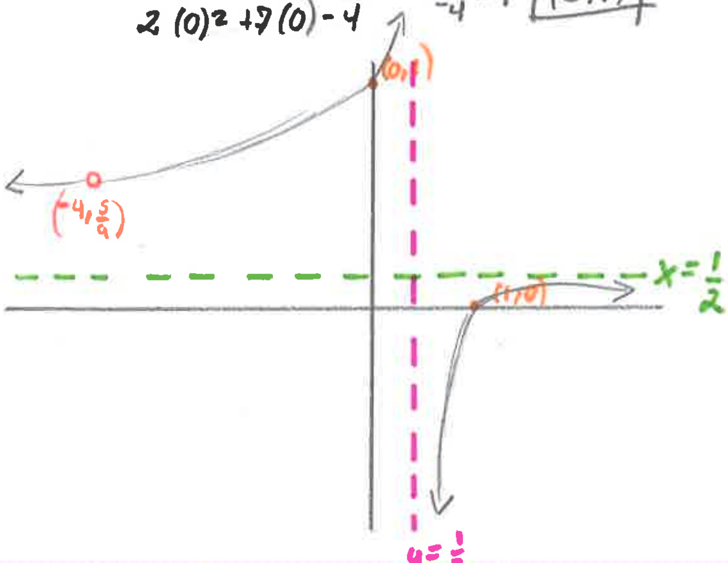
Chapter 6: Rational Functions

- horizontal asymptote: highest exponent value as a fraction
- vertical asymptote: factored denomonator (not including holes)
- hole: values of any numerator + denomenator that cancel out (plug back in w/o hole)
- x-intercept: numerator (without holes)
- y-intercept: plug 0 into x (original equation)
- $\boxed{\text{oblique asymtote}}$ (horizontal)

$\circ\ \frac{x^2}{x^3}\left(\frac{\text{less}}{\text{more}}\right) = y = 0$ $\quad\circ\ \frac{3x^2}{2x^2}\left(\frac{\text{same}}{\text{same}}\right)$ = asymptote at $\frac{3}{2}$ $\quad\circ\ \frac{x^3}{x^2}\left(\frac{\text{more}}{\text{less}}\right)$ = oblique asymtote

ex. $\frac{x^2+3x-4}{2x^2+7x-4} \Rightarrow \frac{(x+4)(x-1)}{(2x-1)(x+4)}$

- HA: $\frac{1x^2}{2x^2} = \frac{1}{2} = \frac{1}{2}$ $\boxed{(x = \frac{1}{2})}$
- VA: $2x-1=0$, $2x=1$, $x=\frac{1}{2}$ $\boxed{(y=\frac{1}{2})}$
- Hole: $x+4=0$, $x=-4$; $\frac{((-4)-1)}{(2(-4)-1)}$; $\frac{-5}{-9} = \frac{5}{9}$ $\boxed{(-4, \frac{5}{9})}$
- x-intercept: $x-1=0$, $x=1$ $\boxed{(1,0)}$
- y-intercept: $\frac{(0)^2+3(0)-4}{2(0)^2+7(0)-4} = \frac{-4}{-4} = 1$ $\boxed{(0,1)}$

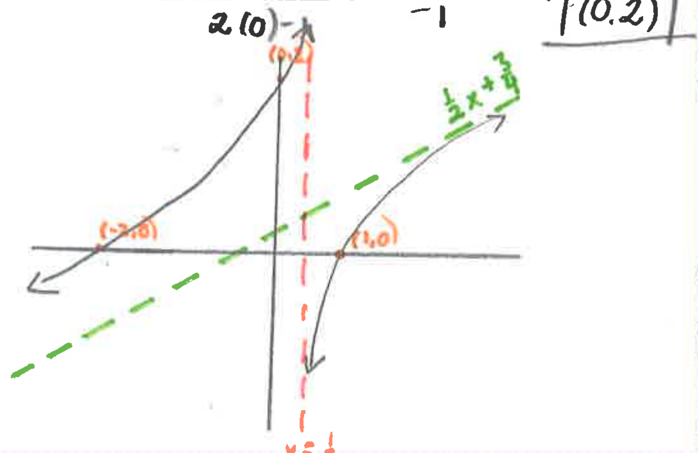


ex. $\frac{(x-1)(x+2)}{(2x-1)} = \frac{x^2+x-2}{2x-1}$

- HA: $\frac{x^2}{x}$ = Oblique asymtote

$$\begin{array}{r|l} & \tfrac{1}{2}x + \tfrac{3}{4} \\ 2x-1 & x^2+x-2 \\ & -(x^2-\tfrac{1}{2}x) \\ & \tfrac{3}{2}x - 2 \\ & -(\tfrac{3}{2}x-2) \\ & 0 \end{array} \Rightarrow \boxed{\tfrac{1}{2}x + 3/4}$$

- VA: $2x-1=0$, $2x=1$, $x=\frac{1}{2}$ $\boxed{x=\frac{1}{2}}$
- Hole: none
- x-intercept: $x-1=0$, $x=1$; $x+2=0$, $x=-2$ $\boxed{(1,0)(-2,0)}$
- y-intercept: $\frac{(0-1)(0+2)}{2(0)-1} = \frac{-1\cdot 2}{-1} = 2$ $\boxed{(0,2)}$



Solving Rational Inequalities

$\frac{x-1}{x+2} > 3$ 1.) make one side 0

$\frac{x-1}{x+2} - 3 > 0$ 2.) make one denominator

$\frac{(x-1) - 3(x+2)}{x+2} > 0$ 3.) combine and simplify

$\frac{-2x-7}{x+2} > 0$ 4.) make positive and switch sign if possible

$\frac{2x+7}{x-2} < 0$ 5. put into point form (x, y) with numerator as x value and denominator as y value

$\boxed{\left(-\frac{7}{2}, -2\right)}$

Solving Rational Equations

$\frac{x+4}{2x-1} = 0$ 1.)

Denominator is undefined

$x = -4$ -numerator

$x \neq \frac{1}{2}$ -denominator

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(Group 3)

SEQUENCES AND SERIES + INDUCTION/DIVISIBILITY

Arithmetic Sequence : $U_n = U_1 + (n-1)d$

* U_1 = 1^{st} term in the sequence
* n = number of terms in the sequence
* d = common difference (+ or −)

↳ example : $-12, -7, -2, 3, \ldots 208$ (+5, +5)

=> $U_1 = -12$

=> $d = +5$

→ $U_n = -12 + (n-1)5$

Geometric Sequence : $U_n = U_1 \cdot r^{n-1}$

* U_1 = 1^{st} term in the sequence
* n = number of terms in the sequence
* r = common ratio (× or ÷)

↳ example : $2, 6, 18, 54 \ldots$ (·3, ·3, ·3)

* $U_1 = 2$
* $r = 3$

=> $U_n = 2 \cdot 3^{n-1}$

· Arithmetic Series : $S_n = \frac{n}{2}(U_1 + U_2)$

· Geometric Series : $S_n = \dfrac{U_1(1-r^n)}{1-r}$

· Infinite Series : $S_\infty = \dfrac{U_1}{1-r}$

Inductive Reasoning in 4 steps

1) Prove the conjecture is true given $n=1$
2) If $n=k$, assume the conjecture is true where $k \in \mathbb{Z}^+$
3) Prove that the conjecture is still true given $n=k+1$
4) State that the conjecture is true for all $n \in \mathbb{Z}^+$

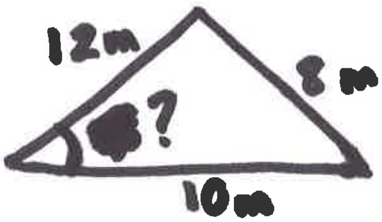
Ch. 10-12 trigonometry

team 5

Period 4

Law of cosines: $a^2 = b^2 + c^2 - 2bc(\cos\theta)$

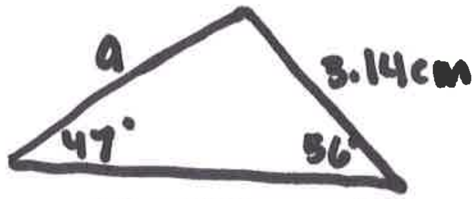
Law of sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$



$8^2 = 12^2 + 10^2 - 2(12)(10)\cos x$

$\cos^{-1}\left(\frac{3}{4}\right) = x$

$\boxed{x = 41.41°}$



$\frac{\sin 47}{3.14} = \frac{\sin 56}{a}$

$\boxed{a = 3.56\text{cm}}$

area of sector: $\frac{1}{2}\theta r^2 = A$

arc length: $l = \theta r$

height of Δ

$h = b \cdot \sin A$

ambiguous case

b, a, h, A	$b > a > h$	2 Δ's
b, a, h, A	$a > b$	1 Δ
b, a, h, A	$h > a$	0 Δs none

the opp. of the given angle is a.

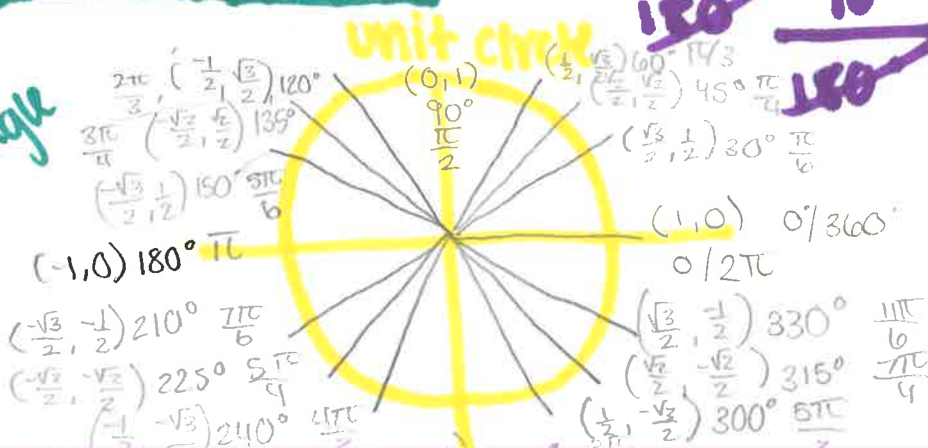
radians $\longrightarrow$ degrees

$\cancel{\pi} \cdot \frac{180°}{\cancel{\pi}} = 180°$

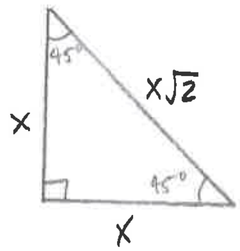
degrees $\longrightarrow$ radians

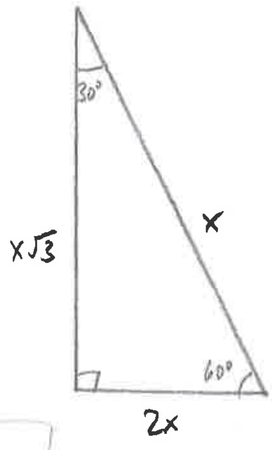
$\cancel{180} \cdot \frac{\pi}{\cancel{180}} = \pi$

unit circle



Special Right Triangles

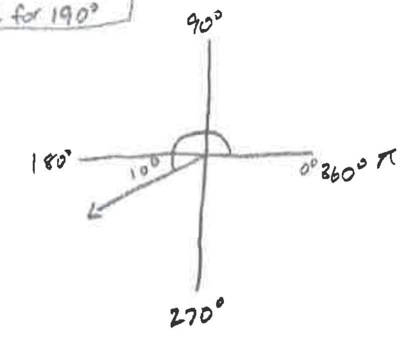


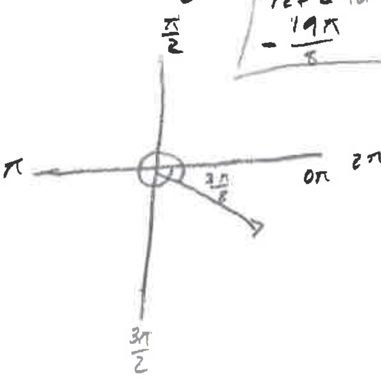


Reference Angles

Draw a ref ∠ for 190°

Draw a ref ∠ for $-\frac{19\pi}{8}$





10°

$\frac{3\pi}{8}$

When drawing a reference angle, if the angle is positive, rotate counter clockwise. If the angle is negative, rotate clockwise.

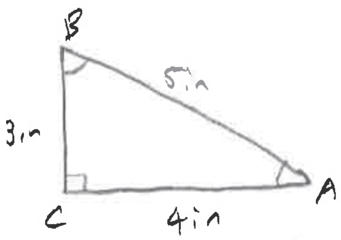
The axes represent either 360° or 2π. You must always draw the reference angle with the x-axis, not the y-axis.

6 Trig Functions

$\sin = \frac{opp}{hyp}$ csc $\text{cosecant} = \frac{1}{\sin} = \frac{hyp}{opp}$

$\cos = \frac{adj}{hyp}$ sec $\text{secant} = \frac{1}{\cos} = \frac{hyp}{adj}$

$\tan = \frac{opp}{adj}$ cot $\text{cotangent} = \frac{1}{\tan} = \frac{adj}{hyp}$



$\sin A = \frac{3}{5}$ $\csc A = \frac{5}{3}$

$\cos A = \frac{4}{5}$ $\sec A = \frac{5}{4}$

$\tan A = \frac{3}{4}$ $\cot A = \frac{4}{3}$

Translations and Transformations

$2x - \frac{\pi}{4}$	x	y	3y − 3
$-\frac{\pi}{4}$	0	0	−3
$\frac{3\pi}{4}$	$\frac{\pi}{2}$	1	0
$\frac{7\pi}{4}$	π	0	−3
$\frac{11\pi}{4}$	$\frac{3\pi}{2}$	−1	−6
$\frac{15\pi}{4}$	2π	0	−3

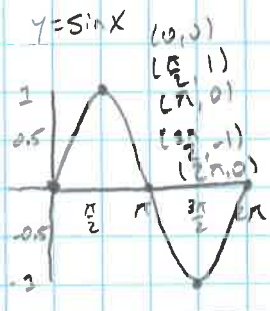
Draw the graph of $y = 3\sin\frac{1}{2}(x - \frac{\pi}{4}) - 3$

amplitude = 3

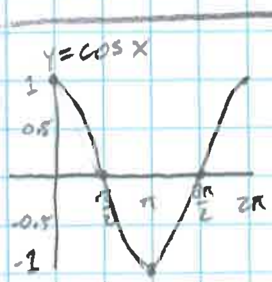
period = $\frac{2\pi}{\frac{1}{2}} = 4\pi$

horizontal shift = right $\frac{\pi}{4}$

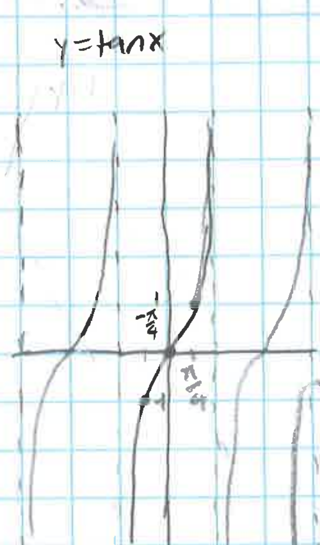
vertical shift = down 3



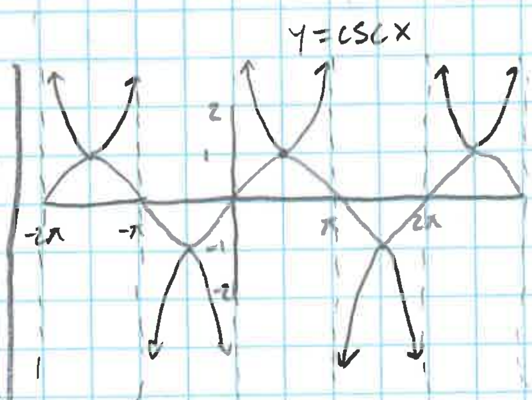
(0, 0)
(π/2, 1)
(π, 0)
(3π/2, −1)
(2π, 0)



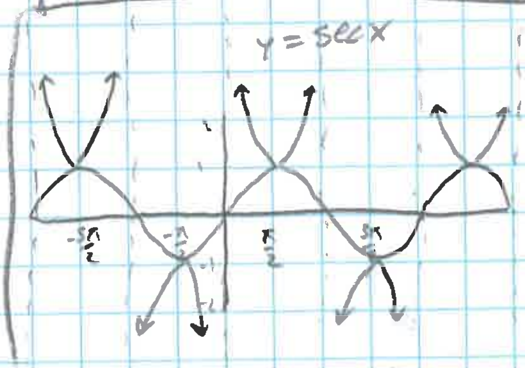
(0, 1)
(π/2, 0)
(π, −1)
(3π/2, 0)
(2π, 1)



(−π/4, −1)
(0, 0)
(π/4, 1)



Draw a normal sine graph



Draw a normal cosine graph.

Chapter 13 Trigonometric Equations and Identities

Trigonometric Identities

$\cot(\theta) = \frac{1}{\tan(\theta)}$ $\qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$

$\csc(\theta) = \frac{1}{\sin(\theta)}$ $\qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$

$\sec\theta = \frac{1}{\cos(\theta)}$

Equations

$\sin^2(\theta) + \cos^2(\theta) = 1$

$\tan^2(\theta) + 1 = \sec^2(\theta)$

$1 + \cot^2(\theta) = \csc^2(\theta)$

Solving Equations

$\tan^2\theta + 3\sec\theta = 9$, Solve for $\sec\theta$

Use trig identity

$\tan^2(\theta) = \sec^2(\theta) - 1$

$\sec^2\theta - 1 + 3\sec\theta = 9$

$\sec^2\theta + 3\sec\theta - 10 = 0$

$(\sec\theta - 5)(\sec\theta + 2) = 0$

$\sec\theta = -2, 5$

Identity Proof

$\frac{\csc x}{\cos x} - \frac{\cos x}{\sin x} = \tan x$

$\frac{\sin x(\csc x)}{\cos x \sin x} - \frac{\cos^2 x}{\cos x \sin x}$

$\frac{1-\cos^2 x}{\cos x \sin x}$

$\frac{\sin^2 x}{\cos x \sin x}$

$\frac{\sin x}{\cos x}$

$\tan x = \tan x$

$\arctan(\frac{1}{6}) + \arctan(\frac{5}{7}) = \frac{\pi}{4}$

$\tan(\arctan\frac{1}{6} + \arctan\frac{5}{7}) = \frac{\tan(\arctan\frac{1}{6}) + \tan(\arctan\frac{5}{7})}{1 - \tan(\arctan\frac{1}{6})\tan(\arctan\frac{5}{7})}$

$= \frac{\frac{1}{6} + \frac{5}{7}}{1 - \frac{1}{6}\cdot\frac{5}{7}}$

$= \frac{\frac{37}{42}}{1 - \frac{5}{42}}$

$= \frac{37/42}{37/42}$

$= 1$

$1 = 1$

$\tan(\pi/4) = 1$

$\arcsin(\theta) = \sin^{-1}(\theta)$

$\arccos\theta = \cos^{-1}(\theta)$

$\arctan\theta = \tan^{-1}(\theta)$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$\sin(A \pm B) = \sin A\cos B \pm \cos A \sin B$

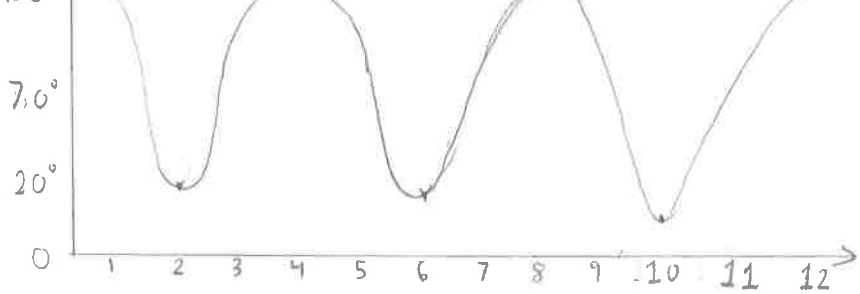
$\cos(A \pm B) = \cos A\cos B \mp \sin A \sin B$

$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

Using Trigonometric Models

A lab sample is being heated and cooled sinusoidally. The first high temp. of 120° occurs at 4 hours. The next low temp. of 20° occurs at 6 hours.

a) Draw a complete sketch of the sinusoid for the interval $0 \le t \le 12$ hours



b) Write an equation in the form $f(t) = a\cos[b(t-h)] + k$

$f(t) = 50\cos(\frac{\pi}{2}t) + 70$

$P = \frac{2\pi}{B}$

$4 = \frac{2\pi}{B}$

$B = \frac{\pi}{2}$

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Ch 8 & 24- Counting and Probability

Permutations- order matters

$_nP_k = \frac{n!}{(n-k)!}$

total # — amount of items in sample

Examples: Orders of a group of people

Combinations- order doesn't matter

$_nC_k = \frac{n!}{n!(n-k)!}$

Examples: how many groups of people

Binomial Expansion

Factorials

$n! = n\cdot(n-1)\cdot(n-2)\ldots$

$5! = 5\times4\times3\times2\times1$

Binomial Theorm:

$\binom{n}{r} a^r (b)^{n-1}$ when $(a+b)^n$

to find the term for a^r

Pascal's Triangle

$(a+b)^1 = 1a+1b$

$(a+b)^2 = 1a^2+2ab+1b^2$

$(a+b)^3 = 1a^3+3a^2b+3ab^2+1b^3$

$(a+b)^4 = 1a^4+4a^3b+6a^2b^2+4ab^3+1b^4$

Probability

Experimental- A coin is flipped 6 times. It shows up with 4 heads and 2 tails. The experimental probability for getting heads is 2/3.

Theoretical- The actual probability for heads is 1/2

Relative Frequency- the fraction of times something occurs

Ex: $\frac{1}{2}$, 50%, 0.5

Independent vs. Dependent

Take 1 and put it back, and pick another

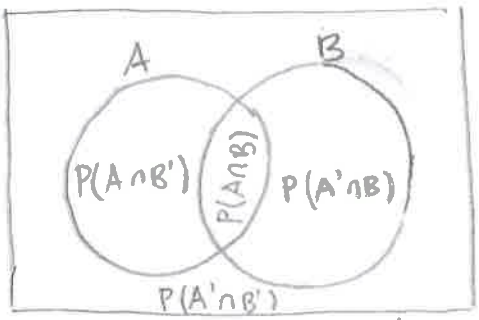
Not affected by other events

Take 1 and don't put it back, then pick another

Affected by other events

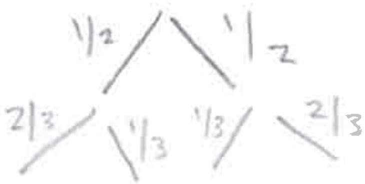
Also is with/without replacement

Venn Diagrams



Everything adds up to 1

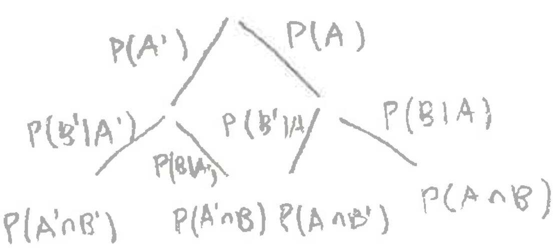
Tree Diagrams



Conditional Probability

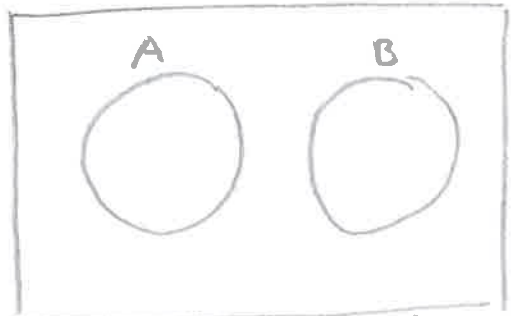
$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Probability of A occuring if B has occured



Mutually Exclusive

$P(A \cap B) = 0$



$P(A \cup B) = P(A) + P(B)$

Complementary Probability

$P(A)$ is complementary to $P(A)'$

$P(A) + P(A') = 1$