Chapter 2

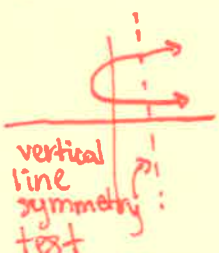
Function vs. Relation:

$f(x)=f(-x) \Rightarrow$ EVEN

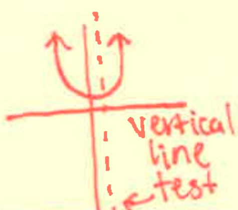
$f(x)=-f(x) \Rightarrow$ ODD

RELATION: Any sets of ordered pairs.

FUNCTION: A set of ordered pairs where there is only one y-value for each x-value.



NOT A FUNCION



FUNCTION

Both are relations

Domain/Range:

Domain: range of all x-values in the function.

Range: range of all y-values in the function.

$p(x)=3+\sqrt{x^2+3x-10}$

$p(x)=3+\sqrt{(x+5)(x-2)}$

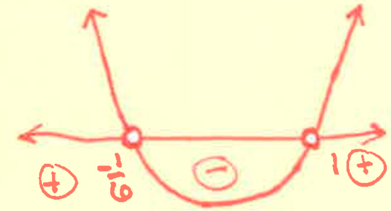
$\boxed{\begin{array}{l} D: (-\infty,-5] \cup [2,\infty) \\ R: (-\infty,\infty) \end{array}}$

Sign Diagrams:

$6x^2-5x-1>0$

$(6x+1)(x-1)>0$

$\boxed{x=(-\infty,-\frac{1}{6}) \cup (1,\infty)}$



Composite Function:

$f(x)=x^2-2$

$g(x)=5x+3$

$(f \circ g)(x)=(5x+3)^2-2$

$\boxed{(f \circ g)x=25x^2+30x+7}$

Piecewise-Defined Functions:

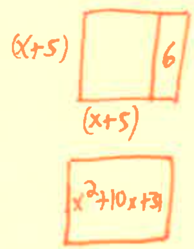
A function defined by multiple sub-functions.

$g(x)=|x-8|$

$\boxed{g(x)=\begin{cases} x-8, & \text{if } x-8\geq 0 \\ -(x-8), & \text{if } x-8<0 \end{cases}}$

Steven Guo
Table 1

Completing the Square



$y = x^2 + 10x + 31$

$y = x^2 + 10x + 25 + 6$

$y = (x+5)^2 + 6$

Maximum and Minimum

$f(x) = x^2 + 10x + 31$

$f(x) = (x+5)^2 + 6$

axis of symmetry $= -b/2a$

$-10/2(1) = -5$

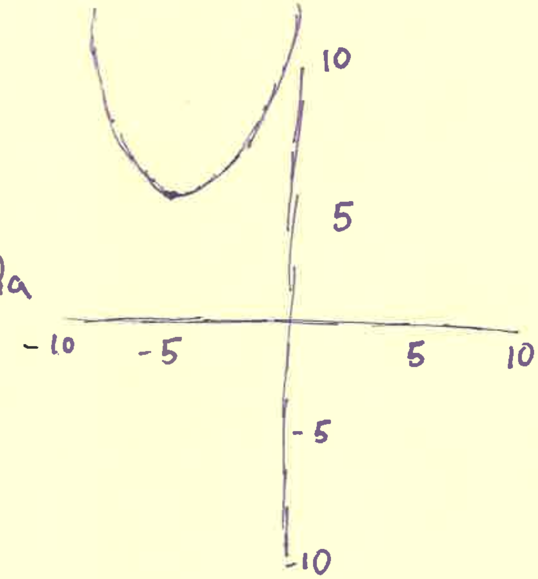
$x = -5$

$f(x) = (-5)^2 + 10(-5) + 31$

$f(x) = 25 - 50 + 31$

$f(x) = -25 + 31$

$f(x) = 6$



Sum and Product of Roots

$y = x^2 + 5x + 6$

$y = (x+3)(x+2)$

roots are $-3, -2$

$(-3) + (-2) = -5$

$(-3)(-2) = 6$

sum $= -b/a$

sum $= -5/1$

sum $= -5$

product $= +c/a$

product $= +6/1$

product $= 6$

Function Notation

$f(x) = 3x + 1$

- This is the way that a function is written

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant

- positive discriminant indicates that the quadratic has two distinct real number solutions.
- when discriminant is zero, it indicates that there is a repeated root
- when negative discriminant, indicates that the solutions are not real numbers.

Graphing

Step 1: Find the vertex by using the equation $x = \frac{-b}{2a}$, then plug into equation

Step 2: Find the y-intercept, where $x=0$ and solve for y

Step 3: Find the x-intercepts. Solve for x by factoring, completing the square or using the quadratic formula.

Step 4: Plot the points found.

Example 1: $y = x^2 + x - 6$

1) $x = \frac{-1}{2(1)} = -\frac{1}{2}$, $y = \left(-\frac{1}{2}\right)^2 + \left(\frac{-1}{2}\right) - 6 = \frac{-25}{4}$

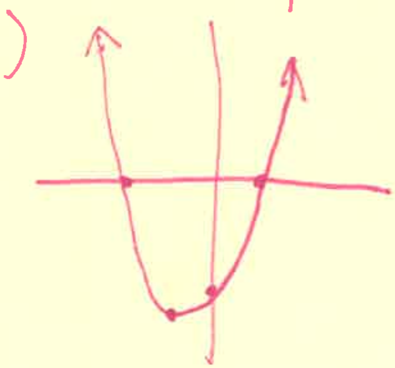
vertex $= \left(-\frac{1}{2}, \frac{-25}{4}\right)$

2) $y = 0^2 + 0 - 6 = -6$ y-int $(0,6)$

3) $x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

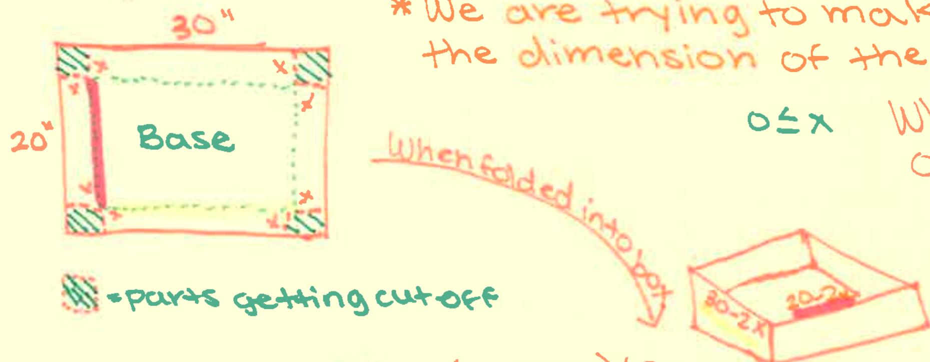
x-int $(-3,0)$ and $(2,0)$

4) 

Optimization

* We are trying to make an open box with the dimension of the base being 20"x30".

$0 \le x$ What are the dimensions of the box being cut?



$V(x) = x(20-2x)(30-2x)$

$V(0) = 0$ $V(10) = 0$

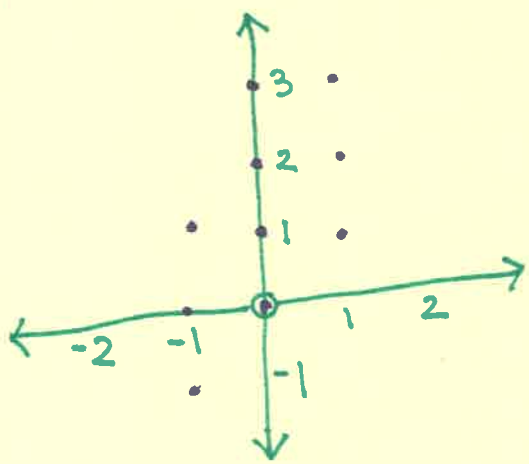
$V(3.89) \approx 1056$

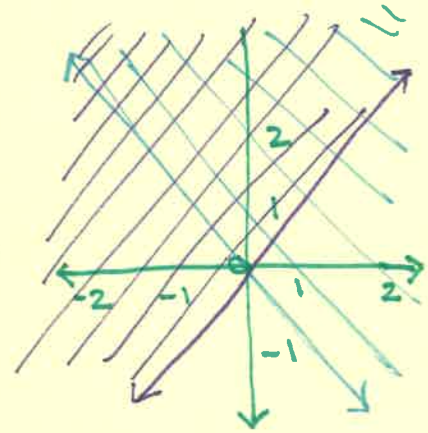
$x \approx 3.85$

$\boxed{3.85" \times 3.85"}$

Inequalities

x	y
0	0
0	1
0	2
0	3
1	1
1	2
1	3
-1	-1
-1	0
-1	1





Chapter 3: Exponential Functions

Vincent Kwok, Saba M, Cory Y.,
Quennie W.

Exponent Rules:

$a^x \cdot a^y = a^{x+y}$ $\quad (ab)^x = a^x b^x$ $\quad a^{-x} = \frac{1}{a^x}$

$a^x \div a^y = a^{x-y}$ $\quad \left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$ $\quad a^{\frac{x}{y}} = \sqrt[y]{a^x}$

$(a^x)^y = a^{xy}$ $\quad a^0 = 1$

Exponential Graphing: $Y = a \times b^{x-c} + d$

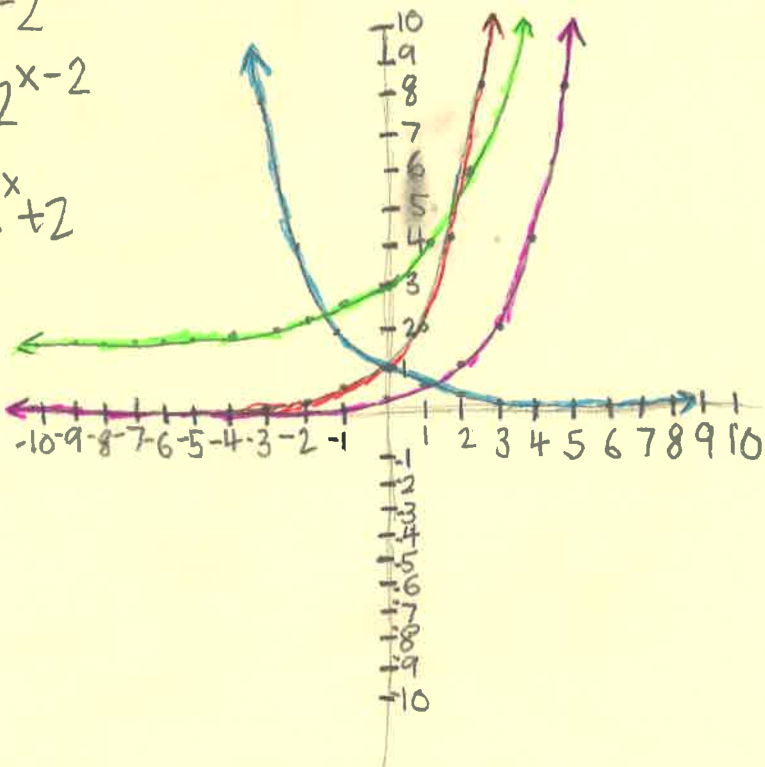
- a controls vertical dilation
- b controls how steep the graph increases/decreases
- C controls horizontal translation
- d controls vertical translation
  horizontal asymptote is $y=d$

$Y = 2^x$

$Y = -2^x$

$Y = 2^{x-2}$

$Y = 2^x + 2$



Algebraic Expansion

$a(b+c) = ab + ac$

$(a+b)(c+d) = ac + ad + bc + bd$

$(a+b)(a-b) = a^2 - b^2$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

Ex. $(2^x+3)(2^x+1)$

$= 2^x \times 2^x + 2^x + 3 \times 2^x + 3$

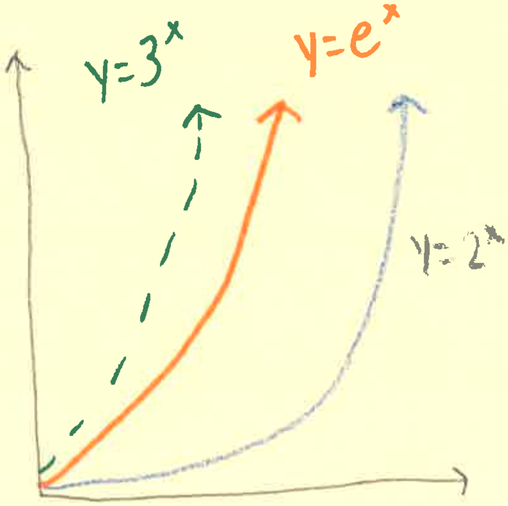
$= 2^{2x} + 4 \times 2^x + 3$

$= 4^x + 2^{2+x} + 3$

The Natual Exponential e^x

$e \approx 2.7183$

Ex.



$e^2 = 7.39$

$\sqrt{e} = e^{\frac{1}{2}}$

$\frac{1}{\sqrt{e}} = e^{-\frac{1}{2}}$

$e^{-1} \approx 0.368$

Exponential Equations

$a^x = a^k$ then $x = k$

Ex. $9^{x-2} = \frac{1}{3}$

$(3^2)^{x-2} = 3^{-1}$

$(3)^{2(x-2)} = 3^{-1}$

$2(x-2) = -1$

$2x - 4 = -1$

$2x = 3$

$x = \frac{3}{2}$

Factorization

Ex. $2^{n+3} + 2^n$

$= 2^n \cdot 2^3 + 2^n$

$= 2^n(2^3 + 1)$

$= 2^n \times 9$

Saba Mir
per. 5
6/12/18

SOLVING EQUATIONS

Equation 1 if $p=\log_b 2$, $q=\log_b 3$, $r=\log_b 5$, write in terms of p & q & r

$\log_b 108$

$$\underset{P}{\log_b 2} + \underset{Q}{\log_b 3} + \underset{R}{\log_b 5} = \log_b(2\cdot3\cdot5)$$
$$= \log_b 30$$

$$2\log_b 2 + 3\log_b 3 = \log_b 2^2 + \log_b 3^3$$
$$= \log_b(2^2 \cdot 3^3)$$
$$= \log_b 4\cdot27 \rightarrow \boxed{\log_b 108}$$

Equation 2 write $\log_n M = 2\log_n b + \log_n c$ without logs

$$\log_n M = \log_n b^2 + \log_n c$$
$$\frac{\log_n M = \log_n (cb^2)}{\log_n}$$
$$\boxed{M = cb^2}$$

Laws of Logarithms

- If A and B are both positive then: $\log A + \log B = \log AB$

$\log A - \log B = \log \frac{A}{B}$

$n \log A = \log A^n$

- In base c where $c \neq 1$, $c > 0$, then: $\log_c A + \log_c B = \log_c AB$

$\log_c A - \log_c B = \log_c \frac{A}{B}$

$n \log_c A = \log_c A^n$

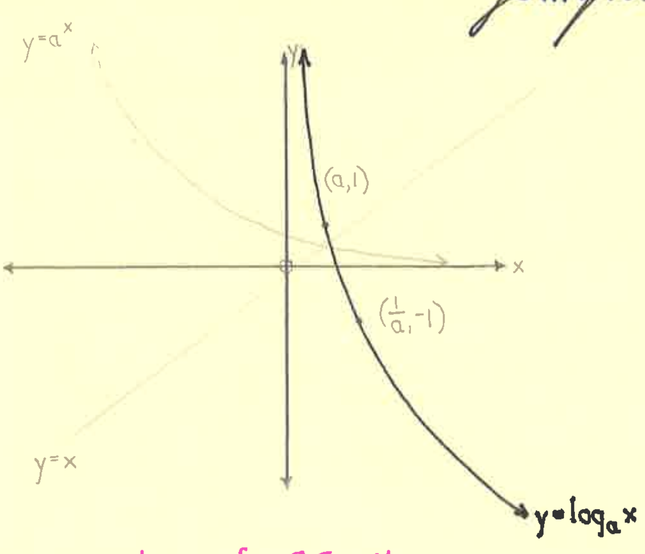
Ex. $2\log 7 - 3\log 2$

$= \log(7^2) - \log(2^3)$

$= \log 49 - \log 8$

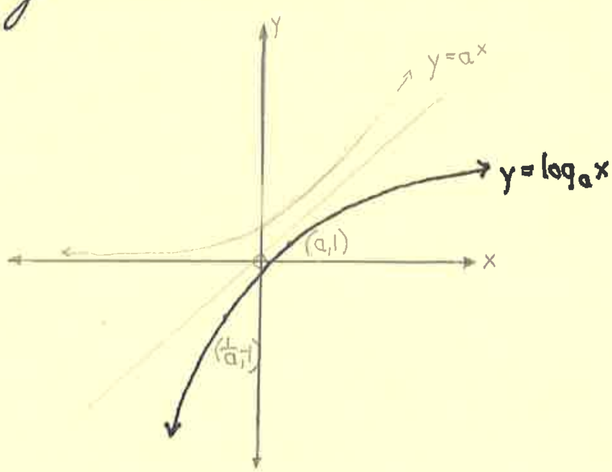
$= \log \frac{49}{8}$

Graphing



$y = \log_a x$ for $0 < a < 1$

$x \to \infty, y \to -\infty$
$x \to 0^+, y \to \infty$



$y = \log_a x$ for $a > 1$

$x \to \infty, y \to \infty$
$x \to 0^+, y \to \infty$

Vertical Asymptote is $x = 0$

$y = \log_a(g(x))$ is defined when $g(x) > 0$

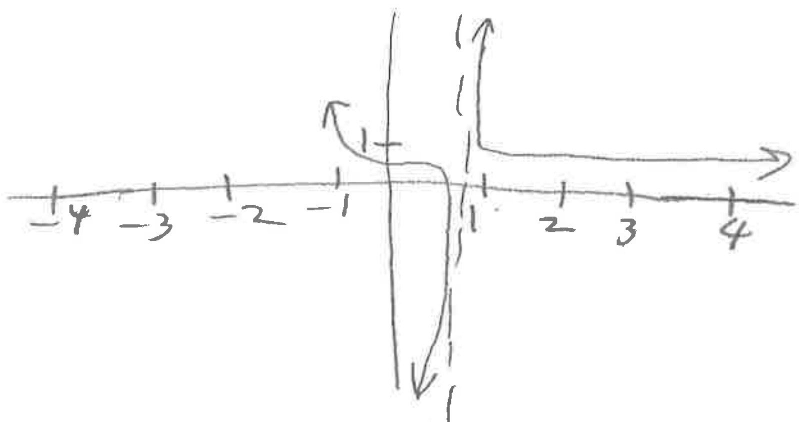
Horizontal Asymptotes

A y-value on a graph which a function approaches but does not actually reach

Example Problem) $f(x)=\frac{2x^3-2}{3x^3-9}$

$$f(x)=\frac{2x^3}{3x^3}$$

$$y=\frac{2}{3}$$

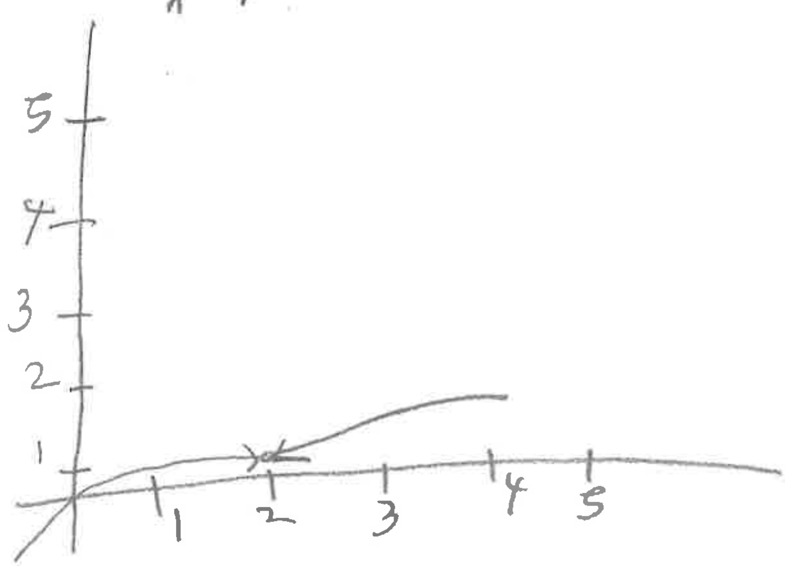


Removable Discontinuities (Holes)

A point in a graph that isn't connected, but can be made by filling in a single point

Example problem) $f(x)=\frac{x^2-2x}{x^2-4}$

$x=2$



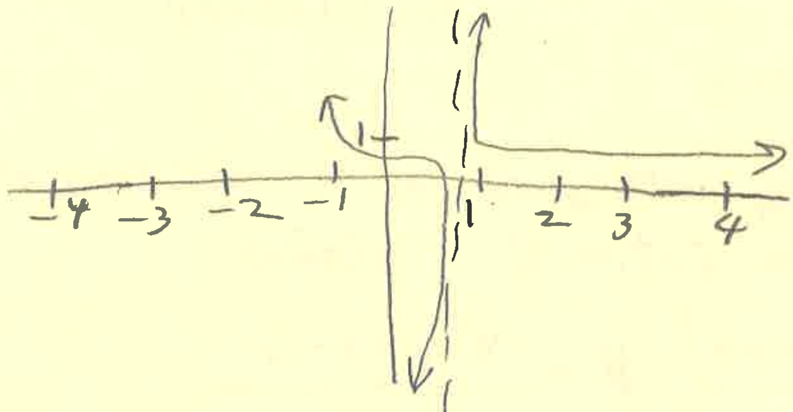
Horizontal Asymptotes

A y-value on a graph which a function approaches but does not actually reach

Example Problem) $f(x) = \dfrac{2x^3 - 2}{3x^3 - 9}$

$$f(x) = \frac{2x^3}{3x^3}$$

$$y = \frac{2}{3}$$



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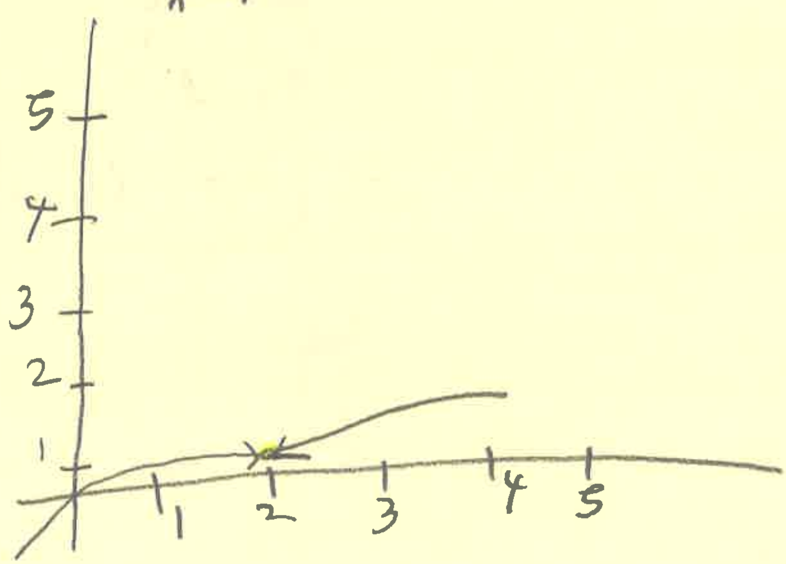


Dhruv Jagannath

Complex Numbers

Complex Numbers: Any number of the form $a+bi$ (a and b are real, $i=\sqrt{-1}$)

Complex

a. $x^2=-4$
$x=\pm\sqrt{-4}$
$x=\pm 2i$

b. $z^2+z+2=0$
$z=\frac{-1\pm\sqrt{1^2-4(1)(2)}}{2}=\frac{-1\pm\sqrt{-7}}{2}=\frac{-1}{2}\pm\frac{\sqrt{7}}{2}i$

c. x^2+4
x^2-4i^2
$(x+2i)(x-2i)$

d. $x^2+9=0$
$x^2-9i^2=0$
$(x+3i)(x-3i)$
$x=\pm 3i$

e. $x^3+2x=0$
$x(x^2+2)=0$
$x(x^2-2i^2)=0$
$x(x+i\sqrt{2})(x-i\sqrt{2})=0$
$x=0 \quad x=\pm i\sqrt{2}$

f. $x^2-4x+13=0$
$x=\frac{4\pm\sqrt{16-4(1)(13)}}{2}$
$x=\frac{4\pm 6i}{2}$
$x=2+3i$ or $2-3i$

Real Polynomials: Polynomials with real coefficients (doesn't mean real (0)i)

a. $x^2-2x+5 \quad \Delta=(-2)^2-4(1)(5)=-16$ Solutions: $x=1+2i$ and $1-2i$

b. $x^2+4 \quad \Delta=0^2-4(1)(4)=-16 \quad x=2i$ and $-2i$ are solutions

$1+2i+1-2i=2 \quad (1+2i)(1-2i)=5$ Roots: $1\pm 2i$ Equation: $a(x^2-2x+5)$

Roots: $\sqrt{2}+i+\sqrt{2}-i=2\sqrt{2} \quad (\sqrt{2}+i)(\sqrt{2}-i)=2+1=3 \quad x^2-2\sqrt{2}x+3=0$

~~Solving~~ Rational Equations:

a. $\frac{2}{x}+\frac{3}{x}-1=5 \quad \frac{5}{x}=6 \quad 5x=6 \quad x=\frac{6}{5}$ or $\frac{1}{2}$

b. $\frac{2}{x^2}+\frac{6-x}{x^2}=8 \quad \frac{8-x}{x^2}=8 \quad 8x^2+x-8=0 \quad \frac{-1\pm\sqrt{(1)^2-4(8)(-8)}}{2(8)}$

$\frac{-1\pm\sqrt{257}}{16}=x$

c. $\frac{k}{k^2}+\frac{k^2-1}{k^2}=3 \quad \frac{k^2+k-1}{k^2}=3 \quad k^2+k-1=3k^2 \quad 2k^2-k+1=0$

$k=\frac{1\pm\sqrt{(-1)^2-4(2)(1)}}{2(2)}=\frac{1\pm i\sqrt{7}}{4}$

Long Division

Sean Steck

Example) Given that $2+i$ is a root of $x^3-6x^2+13x-10=0$ find the other roots

Since $2+i$ is a root, $2-i$ is also a root

1. Multiply the two known roots to get the polynomial to divide by

$$[x-(2+i)][x-(2-i)] = [(x-2)-i][(x-2)+i]$$
$$= x^2-4x+4+1 = x^2-4x+5$$

2. To find the third root, use long division!

Polynomial from step 1 ← x^2-4x+5

$$
\begin{array}{r|l}
 & x-2 \\ \hline
x^2-4x+5 & x^3-6x^2+13x-10 \\
- & (x^3-4x^2+5x) \\ \hline
 & -2x^2+8x-10 \\
- & (-2x^2+8x-10) \\ \hline
 & 0
\end{array}
$$

→ Original polynomial

→ Multiply the divisor by ~~the~~ x, then subtract!

→ Repeat the previous step, this time multiply by -2

→ Since the answer is zero, in this case, $x-2$ is the root

3. Therefore, the three roots are $2+i$, $2-i$, and 2

Synthetic Division

Example) Given that $x=1$ is a root of $2x^3-x^2-2x+1$ find all zeros

1. The equation given is $2x^3-x^2-2x+1$, so first take the coefficients and put them into the synthetic division format and put the 1 from $x=1$ into it

$$
\begin{array}{c|cccc}
1 & 2 & -1 & -2 & 1 \\
 & \downarrow & 2 & 1 & -1 \\ \hline
 & 2 & 1 & -1 & 0
\end{array}
$$

a. Drop the 2

B. Then add the 2 to -1

C. Repeat the process until "0" is reached

2. The results of these coefficients become the coefficients in the quadratic

$$2x^2+x-1=0 \Rightarrow (2x-1)(x+1)=0$$

Factor!

$x=-1$ $x=\frac{1}{2}$

3. Therefore the roots of the equation are -1, 1, and $\frac{1}{2}$

Rational Root Theorem

↳ A theorem that provides a complete list of possible rational roots of the Polynomial equations where all coefficients are integers

1. Find the factors of the first coefficient and the last coefficient
2. Then divide the factors of the last coefficient by the first

Example) $6x^3+8x^2-7x-3$

Factors $\pm1, \pm2, \pm3, \pm6$ — Factors $\pm1, \pm3$

$$\Rightarrow \frac{\text{Factors of } 3}{\text{Factors of } 6} \Rightarrow \pm1, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}, \pm3, \pm\frac{3}{2}$$

POLYNOMIALS

Graphing:

$F(x) = -2(x+4)(x-2)(x+5)$

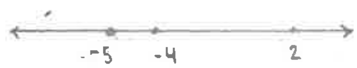
① find the zeros!

$x+4=0 \Rightarrow x=-4$

$x-2=0 \Rightarrow x=2$

$x+5=0 \Rightarrow x=-5$

② put it on a number line!



③ Pop-in some values to test positivity

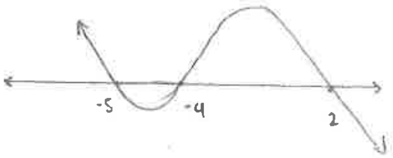
$F(-10) = -2(-)(-)(-)$

$- \cdot - \cdot - \cdot - = +$

$F(3) = -2(+)(+)(+)$

$- \cdot + \cdot + \cdot + = -$

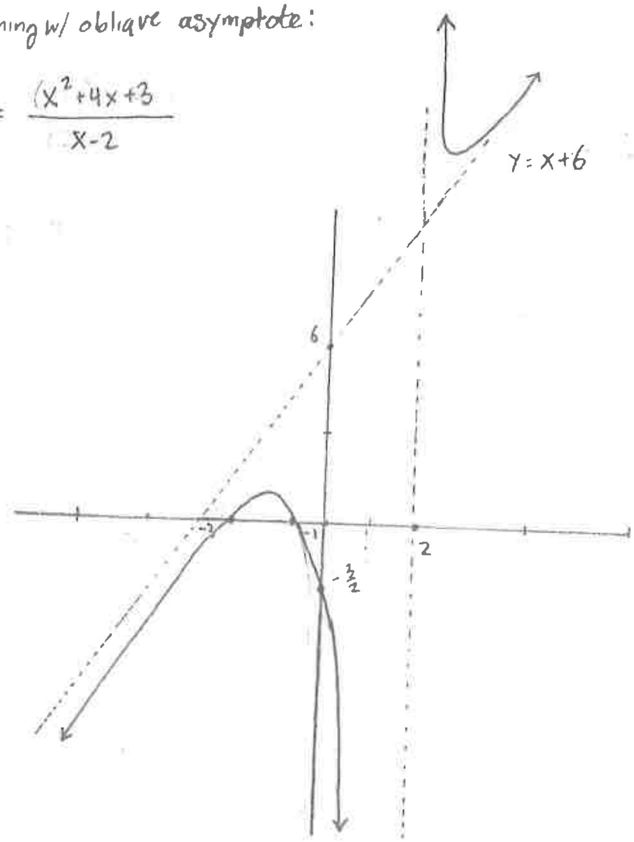
④ Overlay onto number line



RATIONALS

Graphing w/ oblique asymptote:

$f(x) = \dfrac{x^2+4x+3}{x-2}$



vertical asymptote $= 2$

Oblique asymptote:

$$\begin{array}{r|rrr} 2 & 1 & 4 & 3 \\ & & 2 & 12 \\ \hline & 1 & 6 & 15 \end{array}$$

← divide top by bottom

$f(x) = (x+6) + \dfrac{15}{x-2} = x+6$

holes: none

X int: $x^2+4x+3 = 0$

$(x+3)(x+1) = 0$

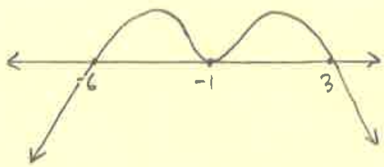
$x = \{-3, -1\}$

Y int: $y = -\frac{3}{2}$, $\rightarrow (0, -\frac{3}{2})$

polynomials: additional

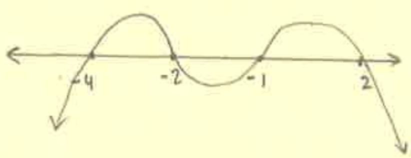
1. $f(x) = -(x+1)^2(x-3)(x+6)$

pro tip: if there is a squared root, the graph will "bounce" on the line! cool!

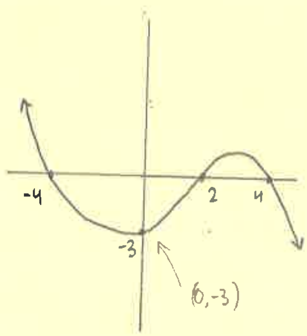


2. $f(x) = (2-x)(x+1)(x+2)(x+4)$

$(-x+2)$



3. Find equation of



$-3 = a(0-4)(0-2)(0+4)$

$a = -\frac{3}{32}$

$f(x) = -\frac{3}{32}(x-4)(x-2)(x+4)$

Oblique asymptotes: additional

1. $f(x) = \dfrac{(x-1)(x+2)}{(2x-1)}$

$$\begin{array}{r|l} & \frac{1}{2}x + \frac{3}{4} \\ 2x-1 & x^2 + x - 2 \\ & \underline{x^2 - \frac{1}{2}x} \\ & \frac{3}{2}x - 2 \\ & \underline{\frac{3}{2}x - 2} \\ & 0 \end{array}$$

oblique asymptote: $\frac{1}{2}x + \frac{3}{4}$

End Behavior

Matteo Schulz

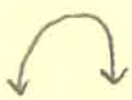
positive = up first
negative = down first
even degree = same second
odd degree = opposite second

positive, odd

$4x^5$ =

negative, even

$-3x^6$ =



negative, odd

$-2x^3$ =

positive, even

$7x^6$ =

Factoring

1. $6x^2-13x-5$

   -13

   $-15 \quad 2$

   -30

   $(6x^2-15x)+(2x-5)$

   $\Rightarrow 3x(2x-5)+(2x-5)$

   $\Rightarrow (3x+1)(2x-5)$

2. $4x^3+x^2-4x-1$ factor out the x^2

   $\Rightarrow x^2(4x+1)-(4x+1)$

   $\Rightarrow (x^2-1)(4x+1)$

   $\Rightarrow (x+1)(x-1)(4x+1)$

chapter 7:

ARITHMETIC SEQUENCES

an arithmetic series is a sequence of numbers with a common and constant difference. (pattern)

$$\boxed{U_n = U_1 + (n-1)d}$$

Ex.

a) $3, 5, 7, \ldots 21$ how many terms?

$\underbrace{3,5}_{2}\underbrace{,7}_{2}$

$21 = 3 + (n-1)2$

$18 = (n-1)2$

$9 = n-1$

$\boxed{n = 8}$

b) $15, 5x-4, \frac{x}{5}$ (consecutive)

$15 - (5x-4) = 5x - 4 - \frac{x}{5}$

$15 - 5x + 4 = 5x - 4 - \frac{x}{5}$

$23 = 10x - \frac{x}{5}$

$115 = 50x - x$

$115 = 49x \longrightarrow \boxed{x = \frac{115}{49}}$

SIGMA & SUMMATION (series)

$$\boxed{S_n = \frac{n}{2}(U_1 + U_n)}$$

Ex.

a) $3 + 7 + 11 \ldots + 159$

$\underbrace{3+7}_{4}\underbrace{+11}_{4}$ $n = ?$

$d = 4$
$U_1 = 3$
$U_n = 159$

$159 = 3 + (n-1)4$

$156 = (n-1)4$

$39 = n-1$

$n = 40$

$S_n = \frac{40}{2}(3+159)$

$= 20(162)$

$= 3240$

$$\sum_{n=1}^{40} 3 + (n-1)4 = 3240$$

b) $$\sum_{n=1}^{\#\text{ of terms}} U_n + (n-1)d = \text{sum}$$

bonus!

The sum of the first 20 terms of an arithmetic sequence is 1180. The 6^{th} term is 32. what is U_1 and d?

$U_6 = U_1 + (6-1)d$

$32 = U_1 + 5d$

$U_1 = 32 - 5d$

$1180 = \frac{20}{2}(U_1 + U_{20})$

$118 = U_1 + U_{20}$

$118 = U_1 + U_1 + 19d$

subsitution!

$118 = 32 - 5d + 32 - 5d + 19d$

$118 = 64 - 10d + 19d$

$54 = 9d \rightarrow \boxed{d = 6}$

$U_1 = U_1$

$U_2 = U_1 + d$

$U_3 = U_1 + 2d$

...

$U_{20} = U_1 + 19d$

$U_1 = 32 - 5d$

$U_1 = 32 - 5(6)$

$\boxed{U_1 = 2}$

GEOMETRIC SERIES
(Chapter 7)

A geometric series has a common ratio (known as "r").

Formula for n^{th} term: $\underbrace{U_n}_{n^{th}\text{ term}} = \overbrace{U_1}^{1^{st}\text{ term}} \underbrace{r}_{\text{common ratio}}^{n-1}$

Find the 8^{th} term:

15, 37.5, 93.75 ...

1) find r $\frac{37.5}{15} = 2.5$ or $\frac{93.75}{37.5} = 2.5$

2) Formula $U_n = 15(2.5)^{n-1}$

3) Solve $U_8 = 15(2.5)^{8-1} = \boxed{9155.27}$

OR

$U_n = \underbrace{U_0}_{0^{th}\text{ term}} r^n$

Sigma Notation:

Formula for the n^{th} term: $\sum_{i=1}^{n} U_1 r^{n-1}$

n ← ending term

i=1 ← Starting term

-Summation: $S_n = \frac{U_1(1-r^n)}{1-r}$

Formula for infinite sequences: $\sum_{i=1}^{\infty} U_1 r^{n-1}$

∞ ← infinity = ending term

-Summation: $S_\infty = \frac{U_1}{1-r}$

U_1

Solve: 54+18+6+2 ...

1) Find r: $\frac{2}{6}$ or $\frac{6}{18}$ or $\frac{18}{54} = \frac{1}{3}$

2) Formula $\frac{54}{1-1/3}$

3) Solve: $U_\infty = \frac{54}{2/3} = \boxed{81}$

INDUCTION

Therina Dokka

arithmetic and geometric AND divisibility

STEP ONE: Show that the statement is true for $n=1$

STEP TWO: Assume that the statement is true for $n=k$, where $k \in Z^+$

STEP THREE: Prove that the statement is true for $n=k+1$

STEP FOUR: The statement is true for $n \in Z^+$

EXAMPLES:

Conjecture: $S_n = 1+5+9 \ldots (4n-3) = n(2n-1)$

1) If $n=1 \rightarrow 4(1)-3 = 1$

$1[2(1)-1] = 1$

$\therefore$ The conjecture is true for $n=1$

2) Assume where $k \in Z^+$ $\Rightarrow 1+5+9 \ldots (4k-3) = k(2k-1)$ is true for $n=k$

3) If $n=k+1 \Rightarrow 1+5+9 \ldots (4k-3) + [4(k+1)-3]$

from step 2 $\rightarrow$ $= k(2k-1) + [4(k+1)-3]$

$= 2k^2 - k + 4k + 4 - 3$

$= 2k^2 + 3k + 1$

$(2k+1)(k+1)$

$(k+1)[2(k+1)-1]$

4) $\therefore$ The conjecture is true for $n \in Z^+$

Ex 2

Conjecture $\sum_{i=1}^{n} \frac{1}{(3i-1)(3i+2)} = \frac{n}{6n+4}$

1) If $n=1 \Rightarrow \frac{1}{[3(1)-1][3(1)+2]} = \frac{1}{10}$

$\frac{1}{6(1)+4} = \frac{1}{10} \Rightarrow n(1)$ is true

2) If $n=k$ where $k \in Z^+ \Rightarrow \sum_{i=1}^{k} \left(\frac{1}{(3i-1)(3i+2)}\right) = \frac{k}{6k+4}$ is true

3) If $n=k+1 \Rightarrow \sum_{i=1}^{k+1} \left(\frac{1}{(3i-1)(3i+2)}\right) = \frac{k}{6k+4} + \frac{1}{[3(k+1)-1][3(k+1)+2]}$

$= \frac{k}{6k+4} + \frac{1}{(3k+3-1)(3k+3+2)}$

$= \frac{k}{6k+4} + \frac{1}{(3k+2)(3k+5)}$

$= \frac{k}{2(3k+2)} + \frac{1}{(3k+2)(3k+5)}$

$= \frac{(3k+5)(k)}{2(3k+5)(3k+2)} + \frac{2}{2(3k+2)(3k+5)}$

$= \frac{3k^2+5k+2}{2(3k+2)(3k+5)}$

$= \frac{(3k+2)(k+1)}{2(3k+2)(3k+5)}$

$= \frac{k+1}{6k+10}$

4) $\therefore$ The conjecture is true for $n \in Z^+$

Ashley C.
Arnuv P.
Zach Z.
Brayden A.

Chapter 8 - Terms

① Permutations

- Order matters

* $_nP_r = \dfrac{n!}{(n-r)!}$

② Combinations

- Order does not matter

* $_nC_r = \dfrac{n!}{(n-r)!\,r!}$

③ Factorials

- Product of an integer and all integers below it.

Ex: $n! = n(n-1)(n-2)(n-3) \ldots$ etc.

or

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

④ Binomial Expansion

* $(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} (a^{n-r})(b^r)$

Example Problems:

① ${}_5P_3 = x$

$$x = \frac{5!}{(5-3)!}$$

$$= \frac{5!}{2!}$$

$$= \frac{5\cdot4\cdot3\cdot\cancel{2!}}{\cancel{2!}}$$

$$= \boxed{60}$$

② ${}_{18}C_5 = Z$

$$Z = \frac{18!}{(18-5)!\,5!}$$

$$= \frac{18!}{13!\,5!}$$

$$= \frac{18\cdot17\cdot16\cdot15\cdot14\cdot\cancel{13!}}{\cancel{13!}\;5!}$$

$$= \frac{18\cdot17\cdot16\cdot\cancel{15}\cdot14}{\cancel{5}\cdot4\cdot\cancel{3}\cdot2\cdot1}$$

$$= \frac{68544}{8}$$

$$= \boxed{8568}$$

③ $7! = y$

$$y = 7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1$$

$$= \boxed{5040}$$

④ What is coefficient of x^6 in $\left(x^2+\frac{4}{x}\right)^{12}$?

$$\binom{12}{r}(x^2)^{12-r}(4(x^{-1}))^r = A\cdot x^6$$

$$x^{2(12-r)}\,x^{-1(r)} = x^6$$

$$2(12-r) - r = 6$$

$$24-2r-r=6$$

$$24-3r=6$$

$$18=3r$$

$$r=6$$

$$({}_{12}C_6)(4)^6 = A$$

$$\boxed{A = 3784704}$$

CHAPTER 24 - TERMS

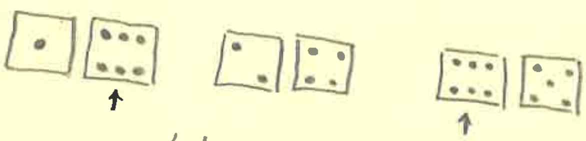
Experimental - what ACTUALLY happened.

Theoretical - what SHOULD happen.

Example:

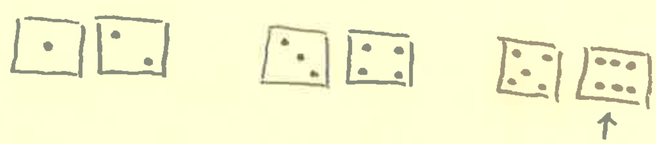
______ probability of rolling a 6 within six rolls (6-sided dice).

a. Experimental



66.6% chance.

b. Theoretical



33.3 chance.

Relative Frequency - $\frac{\text{how often an event occurs}}{\text{\# of potential outcomes}}$. ← numerator = "frequency"

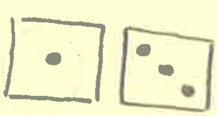
Example: the basketball team wins 9 games out of 12. The frequency of them winning is 9. Their relative frequency is 75%.

Independent - the result of an event has no effect on the next.

Dependent - the result of an event impacts the next.

Example:

a. Independent



(the rolling of one has no effect on the next roll).

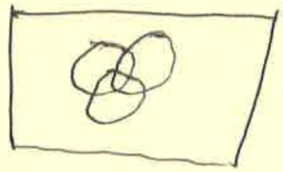
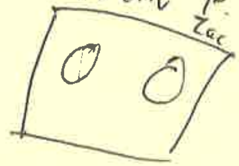
b. Dependent

(one marble gets taken out of the bag).

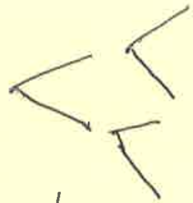
Zach Z, Brayden A., Ashley C., Arnur P.

ch 24 table #7 6/12/18

Per 5
Brayden Allen
Ashley C.
Arnav P.
Zac Z.

Venn Diagrams:   

shows data,

Tree diagrams: 

probability of each action occuring given Privious actions

Mutually exclusive:  can only have 1 of the options, they do not intersect

CH 24 PROBABILITY (CONT)

Arnav Peri
P5
Ashley C
Braeden A
Zach Z

COMPLEMENTARY

$P(A)$ $\xrightarrow{0.4}$

$P(A')$ $1 - P(A) =$ $\xrightarrow{0.6}$

complementary (')
not / opposite of what is being said / asked for

CONDITIONAL Prob.

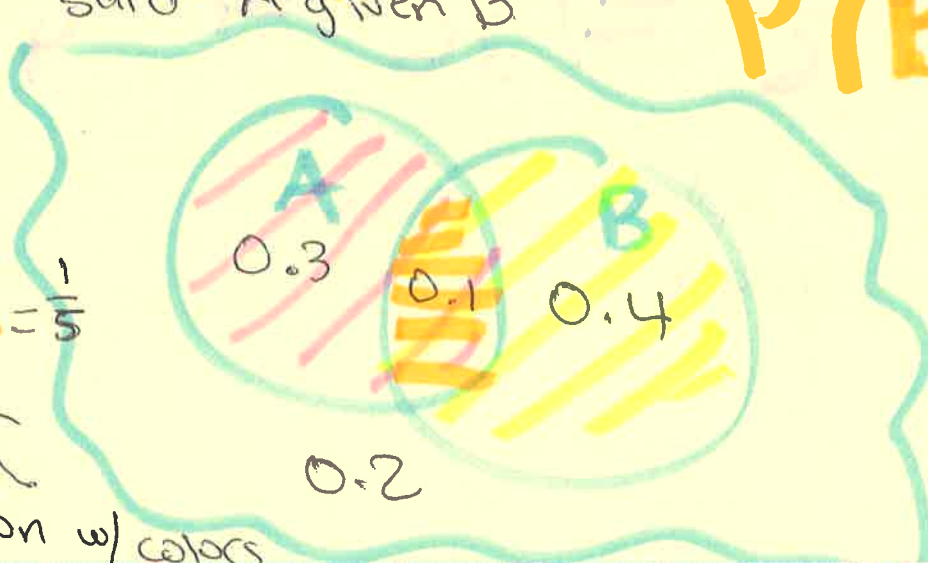
$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

said "A given B"

" "

$$\frac{0.1}{0.5} = \frac{1}{5}$$

equation w/ colors



0.4 A

0.5 B

0.1 $A \cap B$

WITH / OUT REPLACEMENT

3 O MARBLES

3 B MARBLES

NO REPLACEMENT:

P(O), then P(O)

$$\frac{3}{6} * \frac{2}{5} \Rightarrow \frac{6}{30} \Rightarrow \frac{1}{5}$$

one O already taken

REPLACEMENT:

P(B), replace, P(B)

$$\frac{3}{6} * \frac{3}{6} \Rightarrow \frac{9}{36} \Rightarrow \frac{1}{4}$$

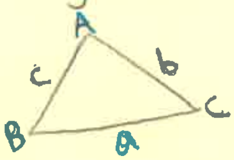
$\frac{3}{6}$ B $\frac{3}{6}$ B, replaced previous marble

Chapter 10-12

Bill Huang

Law of Cosines

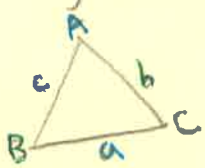
In any $\triangle ABC$



$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

Law of Sines

In any $\triangle ABC$

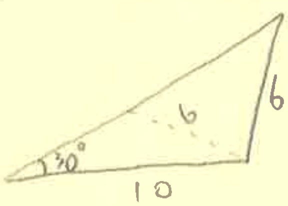


$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{or} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Ambiguous Case

Opposite < Adjacent — 2 triangles

Opposite ≥ Adjacent — 1 triangle or no triangle



$$\frac{\sin C}{c} = \frac{\sin A}{a} \qquad \sin C = \frac{10 \cdot \sin 30}{6}$$

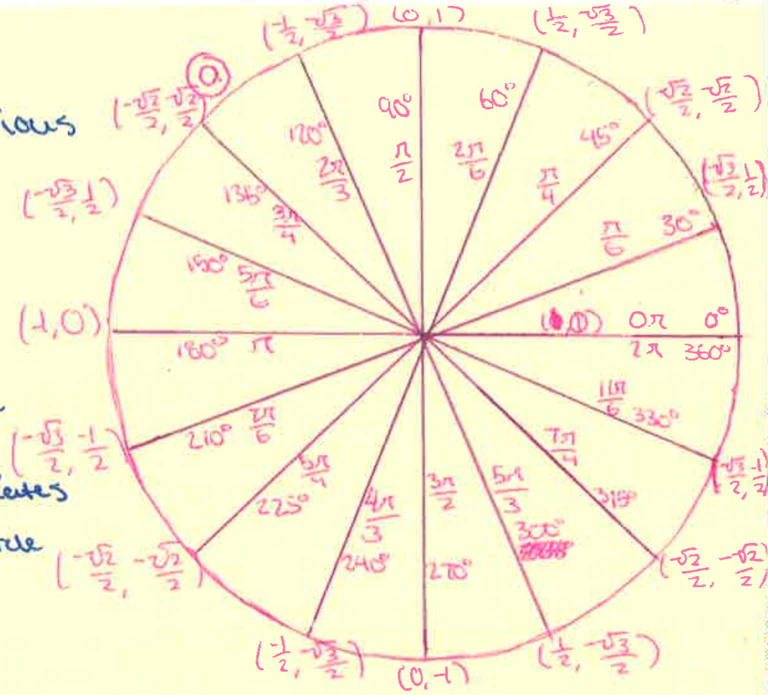
$$\sin C \approx 0.83\overline{3}$$

$C \approx 56.44$, because there are two triangles, the other angle is $180 - \theta = 180 - 56.44 = 123.56$

$$\boxed{C \approx 56.44 \text{ or } 123.56}$$

Unit Circle

- The unit circle allows us to answer various questions regarding sin and cos.



example: (from "a")

1. $\sin 90°$
   $= 1$

   in this situation, sin refers to the y-value of 90° in the unit circle, which has coordinates (0,1)

2. $\cos 135°$
   $= -\frac{\sqrt{2}}{2}$

   in this situation, cos refers to the x-value of 135° in the unit circle, which has coordinates $(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$

*it is also important to remember that unit circle has quadrants, and includes values with π

Arc Length

- To calculate arc length, you divide the given angle by 360°, then multiply that value by the circle's circumference.
  - the base equation is:

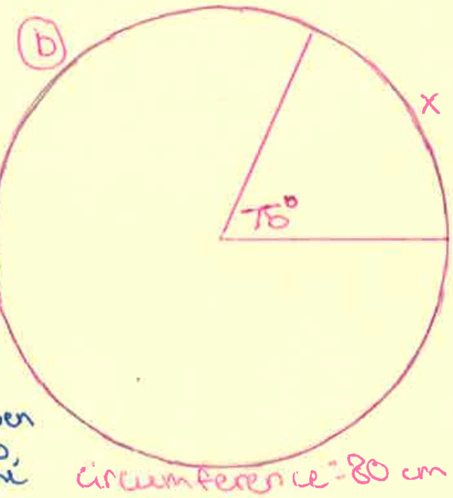
    $\frac{x}{360} \cdot z = y$

    where...

    x: given angle

    z: circumference of entire circle

    y: arc length



example: (from "b")

1. find the value of x

   $\frac{75}{360} \cdot 80 = 16.667$ cm

   in this situation, you plug in the given angle (75) over 360, then multiply by the circumference (80).

Sector Area

- To calculate sector area, you divide the given angle by 360°, then multiply by the total area of the circle
  - the base equation is:

    $\frac{x}{360} \cdot \pi r^2 = y$

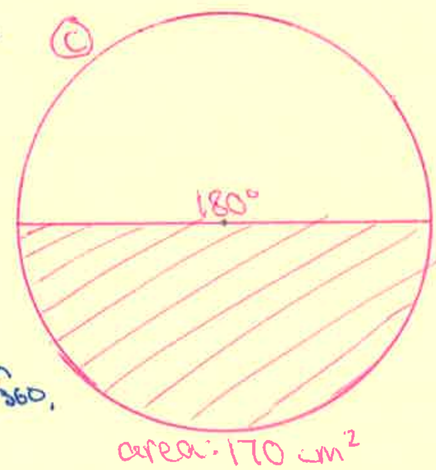
    where...

    x: given angle

    πr^2: area of circle

    r: radius

    y = sector area



example: (from "c")

1. find the area of the shaded region

   $\frac{180}{360} \cdot 170 = 85 cm^2$

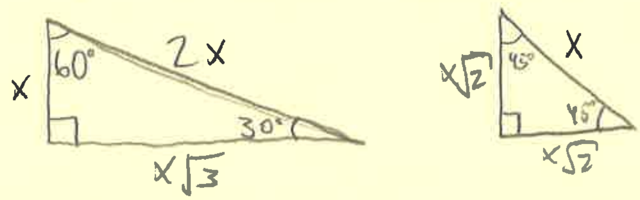
   in this situation, you plug in the given angle (180) over 360, then multiply by the total area (170).

*note that if the units are in radians instead of degrees, you would replace the denominator with 2π, since 360° in a unit circle is equal to 2π.

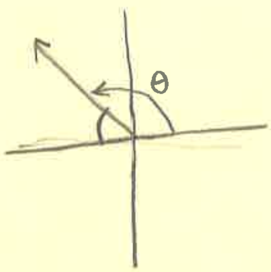
Jason Huang P.5

Ethan Zhu

Special Right Triangles → why $\sin 30° = \frac{1}{2}$



Reference Angles



$\theta = 135°$

reference ∠: 45°

Ex. find reference angle

$\frac{21\pi}{11}$ $\underline{\frac{\pi}{11}}$ $\frac{6\pi}{17}$ $\underline{\frac{6\pi}{17}}$

Trig Functions

$\sin\theta$ $\csc\theta$

$\cos\theta$ $\sec\theta$

$\tan\theta$ $\cot\theta$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\cot\theta = \frac{\cos\theta}{\tan\theta}$

$\sec\theta = \frac{1}{\cos\theta}$ $\csc\theta = \frac{1}{\sin\theta}$

Make sure you can find all 6 given 1

Degrees and Radians

$\pi = 180°$ $2\pi = 360°$

DEG → RAD: $144° \cdot \frac{\pi}{180°} = \frac{144\pi}{180} = \frac{4\pi}{5}$

RAD → DEG: $\frac{4\pi}{5} \cdot \frac{180}{\pi} = \frac{4 \cdot 180}{5} = 144°$

Radians

Junior Patton

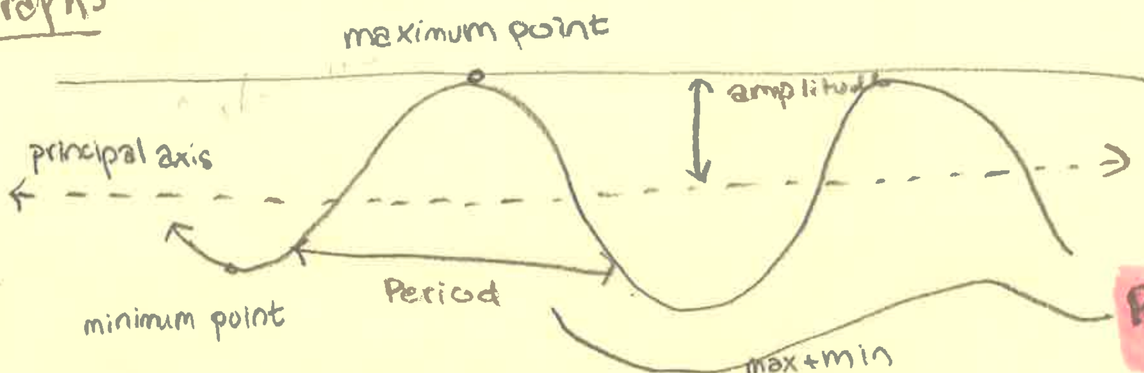
π radians $= 180°$

DEG→RAD $= x° \cdot \frac{\pi}{180}$ $\quad 45° = (45 \cdot \frac{\pi}{180})$ radians $= \frac{\pi}{4}$ radians

RAD→DEG $=$ RAD $\cdot \frac{180}{\pi}$ $\quad 180° = \pi$ radians $\left(\frac{180°}{4}\right) = \frac{\pi}{4}$ radians

$45° = \frac{\pi}{4}$ radians

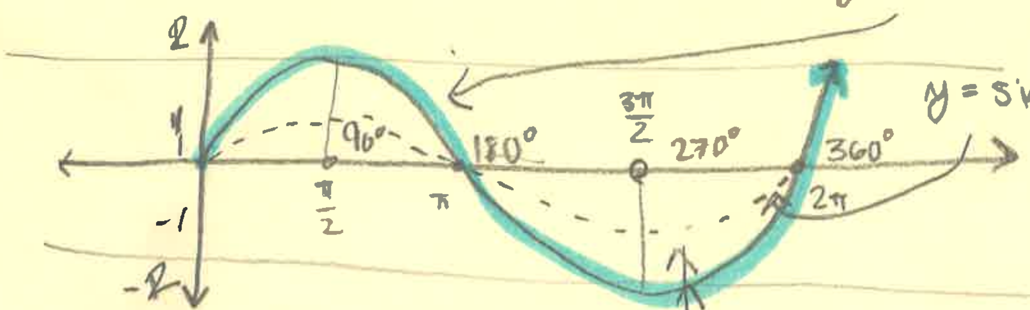
Graphs



$\boxed{\text{Period} = \frac{2\pi}{b}}$

Amplitude $= \frac{\text{max}-\text{min}}{2}$ $\quad$ principal axis $y = \frac{\text{max}+\text{min}}{2}$

$y = 2\sin x$ Basic sine curve



$y = a\sin x$

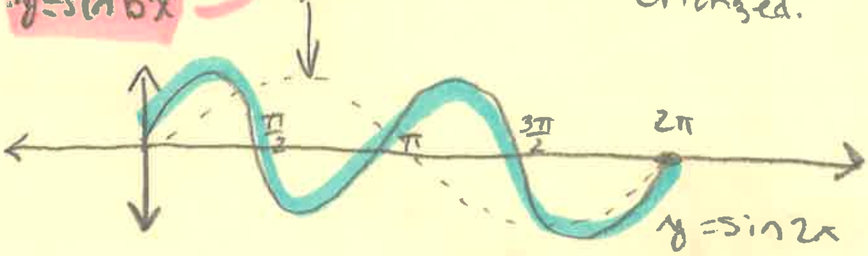
Vertical stretch

vertical stretch is where the amplitude is changed.

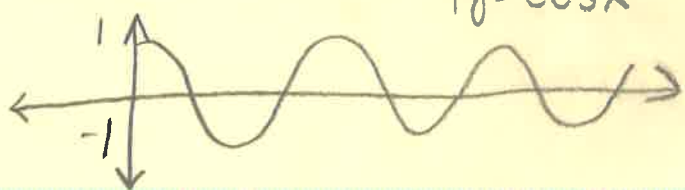
Horizontal stretch

horizontal stretch

$y = \sin bx$



$y = \cos x$



TRANSFORMATIONS

$y = \sin(x - c)$ is horizontal translation

$y = \sin x + d$ = vertical translation

$y = \sin(x - c) + d$ = translation of $y = \sin x$ thru vector $\binom{c}{d}$

CHAPTER 13:

Trevor G.
Ananya G.
Ricky Z.
Julia W.

TRIGONOMETRIC EQUATIONS & IDENTITIES

#1: Find values of x in $(0, 2\pi]$.

a. $5\sin x - 1 = 2\cos^2 x$

$\cos^2 x + \sin^2 x = 1 \Rightarrow \cos^2 x = 1 - \sin^2 x$

$5\sin x - 1 = 2(1 - \sin^2 x) = 2 - 2\sin^2 x$

$2\sin^2 x + 5\sin x - 3 = 0$

$(2\sin x - 1)(\sin x + 3) = 0$

$2\sin x - 1 = 0$ $\quad$ $\sin x + 3 = 0$

$\sin x = \frac{1}{2}$ $\quad$ $\cancel{\sin x = -3}$

no solution

$\boxed{x = \frac{\pi}{6} \ \& \ \frac{5\pi}{6}}$

b. $4\cos^2 x - 3 = 0$

$\cos^2 x = \frac{3}{4}$

$(\cos x)^2 = \frac{3}{4}$

$\cos x = \sqrt{\frac{3}{4}}$

$\cos x = \pm\frac{\sqrt{3}}{2}$

$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$

#3: Find the value of $\tan\left(\frac{\pi}{12}\right)$.

$\tan\left(\frac{\pi}{12}\right) = \tan\left(\frac{\pi}{4} - \frac{\pi}{6}\right) =$

$$\frac{\tan\frac{\pi}{4} - \tan\frac{\pi}{6}}{1 + \tan\frac{\pi}{4}\cdot\tan\frac{\pi}{6}} = \frac{1 - \left(\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)}{1 + \left(1\cdot\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)} =$$

$$\frac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}} = \frac{\frac{3-\sqrt{3}}{3}}{\frac{3+\sqrt{3}}{3}} = \frac{3-\sqrt{3}}{3+\sqrt{3}} = \boxed{2-\sqrt{3}}$$

#2: Verify the following identity.

$$\frac{\sec x}{\sin x} - \frac{\sin x}{\cos x} = \cot x$$

$$\frac{\left(\frac{1}{\cos x}\right)}{\sin x} - \frac{\sin x}{\cos x} = \frac{\left(\frac{1}{\cos x}\right)\cos x - (\sin x)^2}{\sin x\cdot\cos x} = \frac{1-\sin^2 x}{\sin x\cdot\cos x} = \frac{\cos^2 x}{\sin x\cdot\cos x} =$$

$$\frac{\cos x}{\sin x} = \boxed{\cot x}$$

#6: $\arctan\left(\frac{1}{6}\right) + \arctan\left(\frac{5}{7}\right) = \frac{\pi}{4}$

$$\tan\left[\tan^{-1}\left(\frac{1}{6}\right) + \tan^{-1}\left(\frac{5}{7}\right)\right] = \tan\frac{\pi}{4}$$

$$\frac{\frac{1}{6}+\frac{5}{7}}{1-\frac{1}{6}\cdot\frac{5}{7}} = \frac{\frac{7}{42}+\frac{30}{42}}{1-\frac{5}{42}} = \frac{\frac{37}{42}}{\frac{37}{42}} = 1$$

$\tan\frac{\pi}{4} = 1$

#7: Solve for $\sec\theta$: $\tan^2 x + 3\sec\theta = 9.$

$$1+\tan^2 x = \sec^2 x \Rightarrow \tan^2 x = \sec^2 x - 1$$

$$[-1+\sec^2 x] + 3\sec\theta = 9 \Rightarrow \sec^2 x + 3\sec\theta - 10 = 0.$$

$$(\sec x + 5)(\sec x - 2) = 0$$

$$\boxed{\sec x = -5 \qquad \sec x = 2}$$

$$\frac{1}{\cos x} = -5 \qquad \frac{1}{\cos x} = 2$$

$$\cos x = -\frac{1}{5} \qquad \cos x = \frac{1}{2}$$

possible solutions

Chapter 13 EQUATIONS

13

13 13 13 13 13 13 13

13

13 13

13

Trig Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sec\theta = \frac{1}{\cos\theta}$

$\text{cosec}\,\theta = \frac{1}{\sin\theta}$

Negative Angle Identities

$\sin(-\theta) = -\sin(\theta)$

$\cos(-\theta) = \cos(\theta)$

$\tan(-\theta) = -\tan(\theta)$

Pythagorean Identities

$\cos^2\theta + \sin^2\theta = 1$

$\hookrightarrow \cos^2\theta = 1 - \sin^2\theta$

$\hookrightarrow \sin^2\theta = 1 - \cos^2\theta$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

Cofunction Identities

$\sin(\theta) = \cos\left(\frac{\pi}{2} - \theta\right)$

$\cos(\theta) = \sin\left(\frac{\pi}{2} - \theta\right)$

Additive/Subtractive Identities

$\sin(x \pm y) = \sin x \cos y \mp \sin y \cos x$

$\cos(x \pm y) = \cos x \cos y \mp \sin x \cos y$

$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$

Ex: $\sin(x+y) = \sin x \cos y - \sin y \cos x$

$\sin(x-y) = \sin x \cos y + \sin y \cos x$

Double Angle Identities

$\sin 2x = 2\sin x \cos y$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = 1 - 2\sin^2 x$

$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$

$\cos 2x = 2\cos^2 x - 1$

Half Angle Identities

$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$

$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$

$\tan\frac{x}{2} = \frac{1-\cos x}{\sin x}$

$\tan\frac{x}{2} = \frac{\sin x}{1+\cos x}$

$\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$

$x = \theta$

Ex = $\sin^2(\theta)$ where $\theta = 4$

$\sin^2(\theta) = \sin(4^2) = \sin(16)$

Trevor Goo

Arcsine, Arccosine, & Arctangent

Definition : The "arc" of a function is the inverse of the original function

* can also be written as function^{-1} → $\tan^{-1} x$

$\arcsin(\sin x) = x$ $\arccos(\cos x) = x$

$\arctan(\tan x) = x$

Example :

Prove that . . .

$\arctan\left(\frac{1}{6}\right) + \arctan\left(\frac{5}{7}\right) = \frac{\pi}{4}$

$\tan\left(\tan^{-1}\left(\frac{1}{6}\right)\right) = (x) \tan \longrightarrow \tan x = \frac{1}{6}$

$\tan\left(\tan^{-1}\left(\frac{5}{7}\right)\right) = (y) \tan \longrightarrow \tan y = \frac{5}{7}$

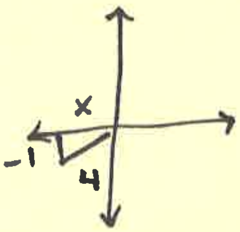
$\tan\left(\frac{\pi}{4}\right) = 1$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \cdot \tan y} = \frac{\frac{1}{6} + \frac{5}{7}}{1 - \frac{1}{6} \cdot \frac{5}{7}} = \frac{\frac{7}{42} + \frac{30}{42}}{1 - \frac{5}{42}}$$

$$= \frac{\frac{37}{42}}{\frac{37}{42}} = 1 \qquad 1 = 1 \ \checkmark$$

#4: The angle θ lies in the third quadrant and $\sin\theta = -\frac{1}{4}$.

a. Find $\sin 2\theta$.



$$(-1)^2 + (x)^2 = (4)^2$$

$$x^2 = 16 - 1 = 15$$

$$x = -\sqrt{15}$$

$$\sin 2\theta = 2\sin\theta\cos\theta =$$

$$2\left(-\frac{1}{4}\right)\left(-\frac{\sqrt{15}}{4}\right) = \frac{2\sqrt{15}}{16} \Rightarrow$$

$$\boxed{\sin 2\theta = \frac{\sqrt{15}}{8}}$$

b. Find $\cos 2\theta$.

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$= \left(-\frac{\sqrt{15}}{4}\right)^2 - \left(-\frac{1}{4}\right)^2$$

$$= \frac{15}{16} - \frac{1}{16}$$

$$= \frac{14}{16}$$

$$\boxed{\cos 2\theta = \frac{7}{8}}$$

#5: Express $\cos\left(\frac{\pi}{3} - x\right)$ in the form $a\cos x + b\sin x$.

a. $\cos\left(\frac{\pi}{3} - x\right) = \cos\frac{\pi}{3}\cos x + \sin\frac{\pi}{3}\sin x =$

$$\boxed{\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x}$$

b. Hence, solve $\cos x + \sqrt{3}\sin x = -1$ for $x \in [0, 2\pi]$.

$$2\left(\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x\right) = -1$$

$$\underbrace{\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}}_{} = -\frac{1}{2}$$

$$\cos\left(\frac{\pi}{3} - x\right) = -\frac{1}{2}$$

$$y = \frac{\pi}{3} - x \Rightarrow \cos y = -\frac{1}{2}$$

$$\boxed{x = \frac{5\pi}{3} \ \& \ \pi}$$

$$y = \cancel{\frac{2\pi}{3}}, \cancel{\frac{4\pi}{3}}, -\frac{4\pi}{3}, -\frac{2\pi}{3}$$

$$0 \le x \le 2\pi$$

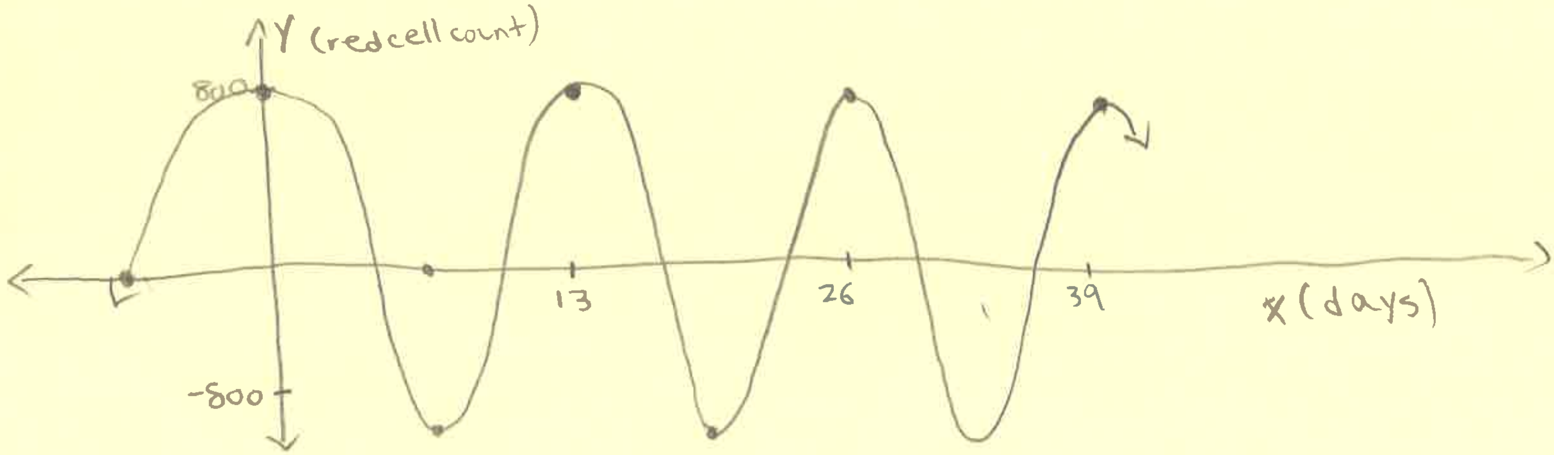
$$\frac{\pi}{3} \le -x + \frac{\pi}{3} \le -2\pi + \frac{\pi}{3}$$

$$y = \frac{-4\pi}{3} \Rightarrow x = \frac{5\pi}{3}$$

$$y = -\frac{2\pi}{3} \Rightarrow x = \pi$$

Sinusoidal Model

Jesus is accused of his crime. He must be stabbed once every 3 weeks. On January 13^{th}, he gets stabbed. At that time, his pain frequency is at 800 Hz. Halfway between punishments, the frequency drops to a low 200. Assume that the frequency varies sinusoidally with the day of the year, x.



Equation: period $= \frac{2\pi}{21}$

amp $= 300$

Right 13

Up 500

$f(x) = 300\cos\left(\frac{2\pi}{21}(x-13)\right)+5$