Chapter 2:
FUNctions

Ryan Fortler.

function vs relation:

Relation: A relation is any set of points which connect 2 variables.

ex:

$x = y^2$ $\quad$ $y = x + 3$

Function: A function, sometimes called a mapping is a relation in which no two different ordered pairs have the same x-coordinates or first component.

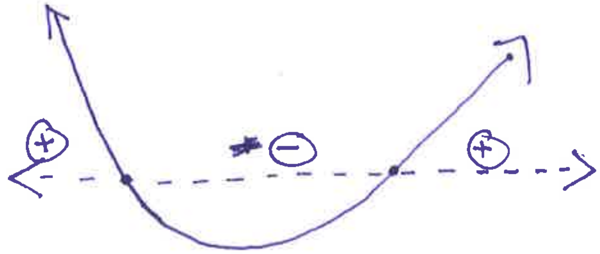
ex:

$f(x) = 2x + 3$

sign diagrams:

contains:

- a horizontal x-axis.
- positive and negative signs.
- zeros of the function.
- values of x where the graph is undefined.



Inequality:

a statement that one expression is greater than, or else greater than or equal to, another.

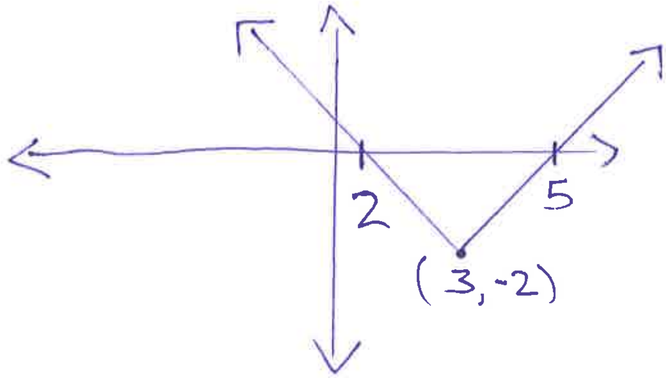
ex:

$2x + 3 > 11 - x$

$\frac{2x-1}{x} \leq \frac{x+3}{5}$

modulus (absolute value)

modulus or absolute value of a real number x is its distance from 0 on the number line. We write the modulus of x as $|x|$.



$y = |x+3| - 2$

Chapter 2: Function Notation, Domain/Range, Inverse, Composite Function

Function Notation

f is used to represent a function →

Ex: f is the function that will convert x into $2x+3$

This function can be written as:

$f : x \longleftrightarrow \underbrace{2x+3}$

function f — such that — x is mapped to $2x+3$

f(x) is read as "f of x"

OR the more common way: $f(x) = 2x+3$ and $y = 2x+3$

Domain and Range

Domain: the set of values of $\boxed{x}$ in the relation

Range: the set of values of $\boxed{y}$ in the relation

Notation:

\# line graph

Set notation: $x \in [a, b]$

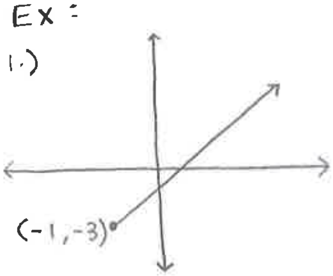
Interval notation: $a \leq x \leq b$

Set notation: $x \in]-\infty, a[\cup]b, \infty[$

Interval notation: $x < a$ or $x > b$

Ex:

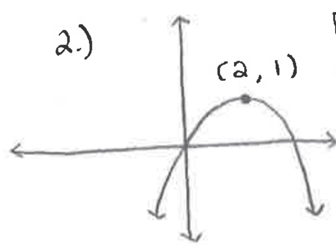
1.)



D: $\{x \mid x \geq -1\}$ or $x \in [-1, \infty[$

R: $\{y \mid y \geq -3\}$ or $y \in [-3, \infty[$

2.)



D: $x \in \mathbb{R}$

R: $y \in]-\infty, 1]$

Inverse

The inverse of $f(x)$ is denoted as $f^{-1}(x)$. It must be a function and must satisfy $(f \circ f^{-1})(x) = x$ and $(f^{-1} \circ f)(x) = x$.

- graph of inverse = graph of original reflected over $y = x$
- domain of f^{-1} = range of f and vice versa

Composite Function

The composite of $f(x)$ and $g(x)$ → $(f \circ g)(x) = f(g(x))$.

Ex: $f(x) = x^4$ and $g(x) = 2x+3$

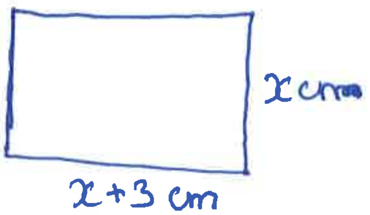
$= f(g(x)) = f(2x+3)$

$\boxed{= (2x+3)^4}$

Emily Park

Problem Solving with Quadratics:

ex. A rectangle has length 3 cm longer than it's width. It's area is $42\,cm^2$. Find its width



$$x(x+3) = 42$$
$$x^2 + 3x - 42 = 0$$

$\therefore$ using technology, roots are $\Rightarrow x = -8.15$ or 5.15

Negative solutions are not possible

$\therefore$ width $= 5.15$ cm

Quadratic Optimisation

ex. Find the maximum value of the function $y = x^2 + x - 3$

$y = x^2 + x - 3$

$a = 1 \quad b = 1 \quad c = -3$

$a > 0 \therefore$ the shape is

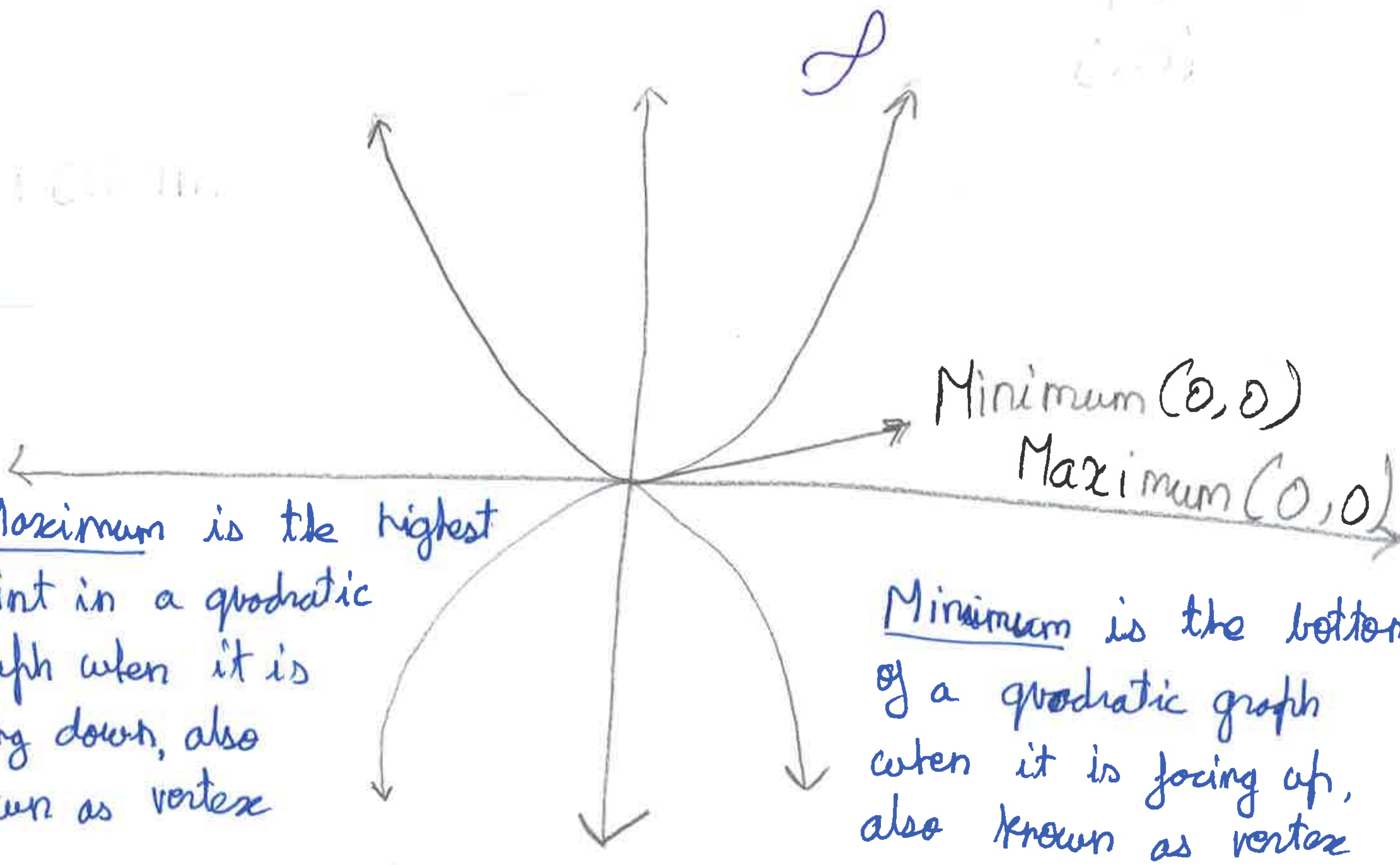
$\therefore$ the minimum value is $x = \frac{-b}{2a}$

$\Rightarrow -\frac{1}{2}$

and $y = \left(\frac{-1}{2}\right)^2 + \left(\frac{-1}{2}\right) - 3$

$= -3\frac{1}{4}$

$\therefore$ The minimum value of y is $-3\frac{1}{4}$, occuring when $x = \frac{-1}{2}$



<u>Maximum</u> is the highest point in a quadratic graph when it is facing down, also known as vertex

<u>Minimum</u> is the bottom of a quadratic graph when it is facing up, also known as vertex

~~D: (0, ∞)~~

~~R: (0, ∞)~~

$\boxed{ax^2 + bx + c}$ $\boxed{\text{roots}: \alpha, \beta}$

Sum: $\frac{-b}{a} = \alpha + \beta$

Product: $\frac{c}{a} = \alpha \cdot \beta$

---

ex. $25x^2 - 20x + 1 = 0$

Sum $= \frac{20}{25} = \frac{4}{5}$

Product $= \frac{1}{25}$

Proof: $\frac{20 \pm \sqrt{400 - 100}}{50}$

$= \frac{20 \pm \sqrt{300}}{50}$

$= \boxed{\frac{2 \pm \sqrt{3}}{5}}$

Sum $= \frac{2 + \sqrt{3}}{5} + \frac{2 - \sqrt{3}}{5}$

$= \frac{4}{5}$

Product $= \left(\frac{2 + \sqrt{3}}{5}\right)\left(\frac{2 - \sqrt{3}}{5}\right)$

Solving equations ...

options...

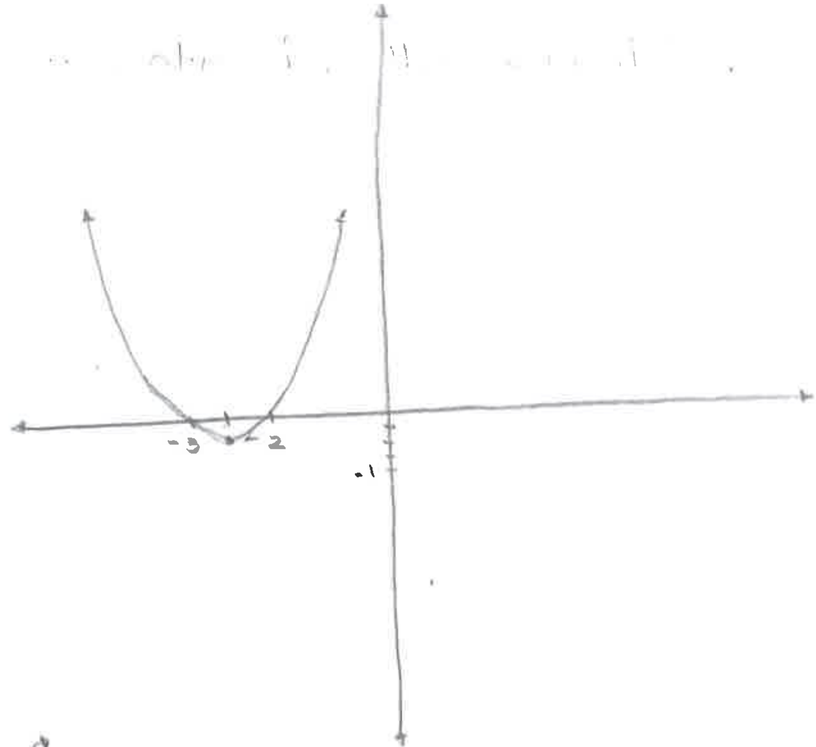
1) Factorize.
2) complete the square
3) quadratic formula.
4) GDC.

example:

$x^2 + 5x + 6 = 0$

$(x+3)(x+2) = 0$

$x = -3$ or $x = -2$



Graphing:

1) Factorize to find roots to find x-intercepts.
2) Plug in 0 for x to find y int.
3) Complete square to find vertex.

example:

$x^2 + 5x + 6$

$(x+3)(x+2)$

$x = -3 \quad x = -2$

$\left(x^2 + 5x + \frac{25}{4}\right) - \frac{25}{4} + \frac{24}{4} = 0$

$\left(x + \frac{5}{2}\right)^2 - \frac{1}{4} = 0$

Discriminant.

1) use $b^2 - 4ac$ to find the discriminant.
2) If the value is $= 0$ there is one x-int.
3) If the value is less than 0, there are no real x-ints.
4) If the value is more than 0, there are 2 real x-intercepts.

$1x^2 + 5x + 6$

a b c

$25 - 4(1)(6)$

$= 25 - 24$

$= 1$

$1 > 0$, so 2 real roots.

$ax^2 + bx + c$

Complete the square:

example:

$x^2 + 8x + 19 = 2.$

$(x^2 + 8x + 16) - 16 + 19 - 2 = 0$

$(x + 4)^2 + 1 = 0$

Steps:

1) Divide everything by a to get

$x^2 + \frac{b}{a}x + \frac{c}{a}$

2) Put parentheses around $x^2 + \frac{b}{a}x$ to get

$(x^2 + \frac{b}{a}x) + \frac{c}{a}$

3) Find the half of $\frac{b}{a}$ and square it to find new c value.

You get:

$(x^2 + \frac{b}{a}x + (\frac{b}{2a})^2)$.

4) Subtract said value from outside parenthesis to maintain equality.

$(x^2 + \frac{b}{a}x + (\frac{b}{2a})^2) - (\frac{b}{2a})^2 + \frac{c}{a}$

5) Factor:

$(x + \frac{b}{2a})^2 - \frac{b^2}{4a^2} + \frac{c}{a}$

Quadratic Formula:

Steps:

1) Establish values a, b and c of equation $(ax^2 + bx + c)$.

2) Plug into quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

3) simplify to find 1 or 2 values of x.

(may be a complex number depending on discriminant).

Example:

$5x^2 + 4x - 2 = 0$

$a = 5, \ b = 4, \ c = -2$

$$x = \frac{-4 \pm \sqrt{16 - 4(5)(-2)}}{2(5)}$$

$$= \frac{-4 \pm \sqrt{16 + 40}}{10}$$

$$= -\frac{2}{5} \pm \frac{\sqrt{56}}{10}$$

$$= -\frac{2}{5} \pm \frac{2\sqrt{14}}{10} = -\frac{2}{5} + \frac{\sqrt{14}}{5} \text{ or } -\frac{2}{5} - \frac{\sqrt{14}}{5}$$

Chapter 3

Exponential Functions

Exponent Rules:

$x^a \cdot x^b = x^{a+b}$

$\{2^3 \cdot 2^5 = 2^8\}$

$8^2 \cdot 16^2 = 2^{3(2)} \cdot 2^{4(2)}$

$2^6 \cdot 2^8 = 2^{14}$

$x^a \div x^b = x^{a-b}$

$* \frac{1}{x^b} = x^{-b}$

$\{3^6 \div 3^4 = 3^2\}$

$\frac{4^5}{8^2} = \frac{2^{2(5)}}{2^{3(2)}}$

$\frac{2^{10}}{2^6} = 2^4$

$(x^a)^b = x^{ab}$

$\{(5^2)^3 = 5^6\}$

$(9^3)^2 = (3^{2(3)})^2$

$(3^6)^2 = 3^{12}$

Foil First Outer Inner Last

$(ax+b)(cx+d) = acx^2 + (ad+bc)x + bd$

First: ac
Outer: ad
Inner: bc
Last: bd

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Alexei Z.

Alexei Zhang

Natural Base e

$\log_e = \ln$

$\downarrow$

$\log_e x = \ln x$

$\ln e = \log_e e = 1$

e is an irrational number used for continuous growth.

Chapter 4: Laws of Logarithms

If A and B are positive:

$\log A + \log B = \log(AB)$

$\log A - \log B = \log\left(\frac{A}{B}\right)$

$n \log A = \log(A^n)$

Works if bases are the same

Ex.

1. $\log 5 + \log 3$
$= \log(5 \cdot 3)$
$= \log(15)$

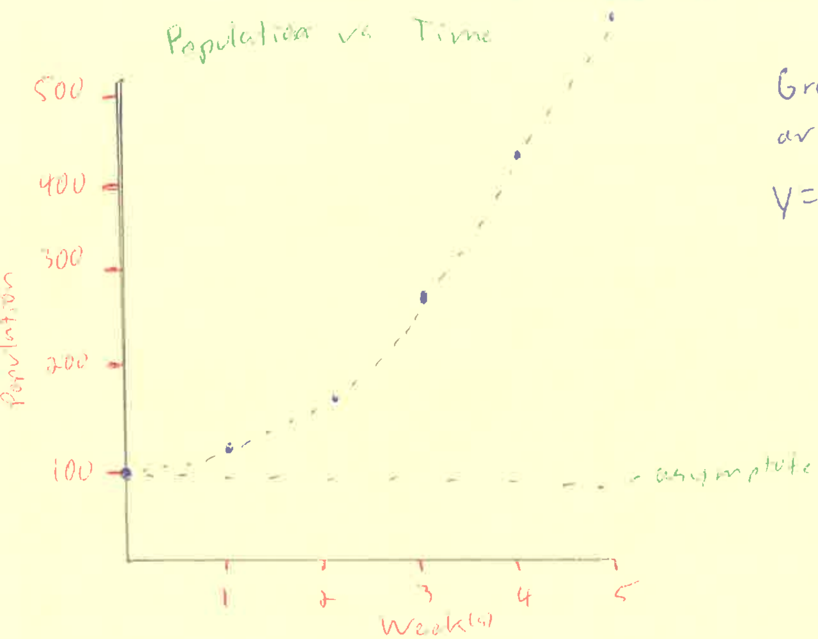
2. $\log_3 24 - \log_3 8$
$= \log_3\left(\frac{24}{8}\right)$
$= \log_3 3 = 1$

3. $\log_2 5 - 1$
$= \log_2 5 - \log_2 2$
$= \log_2\left(\frac{5}{2}\right)$

Emory Jendratzke

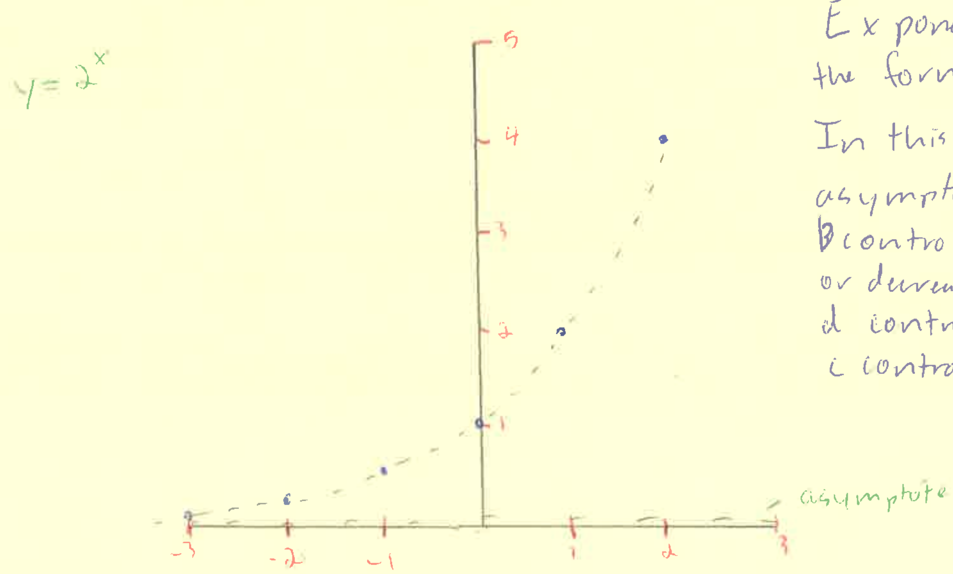
Graphing

Growth and Decay



Growth and decay functions usually are expressed in the form $y = a(b^x)$.

Exponential Functions



Exponential functions come in the form $y = a(b^{x-c}) + d$.

In this form, d is the horizontal asymptote.

b controls how fast the graph increases or decreases.

d controls vertical translation.

c controls horizontal translation.

Anika Ajwani

Chapter 4

Change of Base

Formula: $\log_b x = \dfrac{\log_a x}{\log_a b}$

ex. $\log_2 37 = \dfrac{\log 37}{\log 2} \approx \boxed{5.21}$

Solving Equations

1) $\log\sqrt{10} = x \rightarrow \sqrt{10} = 10^x \rightarrow x = \boxed{\frac{1}{2}}$

2) $\log 16 + 2\log 3 \rightarrow \log 16 + \dfrac{\log 3}{\log 2} \rightarrow \boxed{2.79}$

3) $\log_2 x = -3 \rightarrow x = 2^{-3} \rightarrow \boxed{x = 0.125}$

4) $5^x = 7 \rightarrow \log_5 7 = x \rightarrow \dfrac{\log 7}{\log 5} \rightarrow \boxed{1.21}$

5) $4\ln 2 + 2\ln 3 \rightarrow \ln(2^4) + \ln(3^2) \rightarrow \ln 16 + \ln 9 \rightarrow \boxed{\ln 144}$

polynomials AND rational functions

COMPLEX NUMBERS
- anything which involves the $\sqrt{-1}$, or i

examples: $45i$, $6+7i$, $85i+72$

↳ problem

$\frac{45+6i}{7+5i}$ ← multiply by the conjugate of the denominator

How do we get the conjugate?

↳ take $7+5i$ → flip the sign of the coefficient of i, so $7+5i$ turns into $7-5i$! we have the conjugate

$$\frac{45+6i}{7+5i}\frac{(7-5i)}{(7-5i)} = \frac{345-183i}{49+25} = \frac{345-183i}{74}$$

REAL POLYNOMIALS
- real numbers!

- $\sqrt{-1}$ is not a real polynomial~~s~~
- exist as real numbers
- two negative numbers multiplied together make a positive
  - ↳ two positive numbers multiplied together make a positive

Vishal C., Blake H., Heera R., Matthew G., Rohan A.

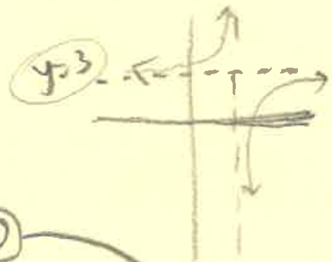
Rational Functions - Asymptotes -

Horizontal Asymptotes

Is there an HA?

$\frac{x^2+4}{x+10}$ = Different Degrees

NO



$\frac{3x^2+3x+4}{x^2-9}$ = yes $\div 3$

$\frac{x-3}{x^2-14} \Rightarrow \frac{0x^2+x-3}{x^2-14} =$ y=0

Oblique Asymptotes

$\frac{x^2-6x+7}{x+5}$ → Divide

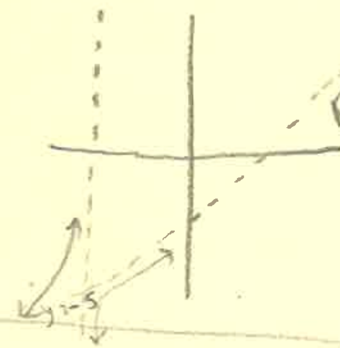
$x+5 \overline{)x^2-6x+7}$ = $x - 11$

$-x^2+5x$

$-11x+7$

$-11x$ ...

OA: $\boxed{x-11=y}$



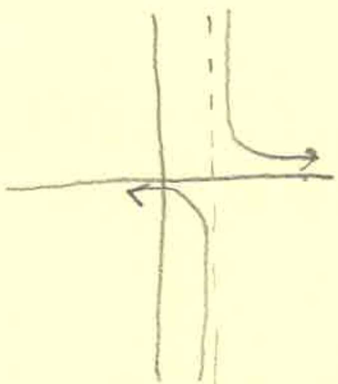
Vertical Asymptotes

$\frac{(x+4)(x+2)}{(x-3)}$

Denominator cannot equal zero

$x-3 \neq 0$
$+3 \quad +3$

$x \neq 3$



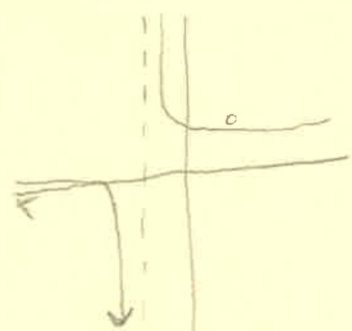
VA↑

Holes

- Find common factor

$\frac{x^2+x-6}{x^2-4} = \frac{(x+3)(x-2)}{(x+2)(x-2)}$ $x-2=0$, $x=2$

$\frac{(x+3)\cancel{(x-2)}}{(x+2)\cancel{(x-2)}} \Rightarrow \frac{x+3}{x+2} \Rightarrow \frac{2+3}{2+2} = \frac{5}{4}$



Hole: $(2, \frac{5}{4})$

Product and Sum of Roots:

There is a very simple method to find sum and product of roots.

① Divide by leading coefficient

$$\frac{9x^3 + 6x^2 - 2x + 9}{9}$$

② If the leading polynomial is an <u>odd</u> power, the sum is ~~negative~~ opposite sign. If it is <u>even</u>, the sum is same sign

2a. The sum is the coefficient of the second highest polynomial

b. The product is the coefficient of x^0 and is always opposite sign

$$x^3 + \underbrace{\tfrac{2}{3}}_{\text{sum}}x^2 - \tfrac{2}{9}x + \underbrace{1}_{\text{prod}} \qquad -1 \cdot \tfrac{2}{3} = \text{sum} \qquad -1 \cdot 1 = \text{product}$$

3) Write down answers

Sum: $-\frac{2}{3}$

product: -1

$\overset{\text{even}}{4x^2 + 3x + 2}$

$$\frac{4x^2 + 3x + 2}{4}$$

$x^2 + \frac{3}{4}x + \frac{1}{2}$

sum: $\frac{3}{4}$

prod: $-\frac{1}{2}$

$\overset{\text{odd}}{10x^5 + 9x^4 + x^2 + 6x}$

$x^5 + \frac{9}{10}x^4 + \frac{x^2}{10} + \frac{3}{5}x$

sum: $-\frac{9}{10}$ prod: $-\frac{3}{5}$

Long Division

$$\frac{x^3-4x^2+2x-3}{x+2}$$

$$\begin{array}{r}
x^2-6x+14 \\
x+2 \overline{\smash{)}\, x^3-4x^2-2x-3} \\
\underline{x^3-2x^2} \\
-6x^2+2x \\
\underline{-6x^2-12x} \\
14x-3 \\
\underline{14x+28} \\
-31
\end{array}$$

$r=-31$

$$x^2-6x+14-\frac{31}{x+2}$$

Synthetic Division

$x+2=0$
$x=-2$

$$\frac{x^3-4x^2+2x-3}{x+2}$$

1. Write the coefficents inside "box"
2. find divisor by setting $x=$
   ex: $x+2=0$ $x=-2$
3. Take the first number inside and carry it down unchanged.
4. multiply the previous carry-down value by the divisor and carry the result into the next exponent
5. add the result with the next exponent and carry down. repeat 4-5

-2	1	-4	2	-3
		-2	12	-28
	1	-6	14	-31

$$x^2-6x+14-\frac{31}{x+2}$$

Rational Root theorem

$$2x^4+x^3-19x^2-9x+9$$

leading coefficient — constant term

Factors of constant term: $\pm1\ \pm3\ \pm9$

Factors of leading coefficient: $\pm1\ \pm2$

Possible values = $\dfrac{\text{Factors of constant term}}{\text{Factors of leading coefficient}}$

$$\frac{\pm1\ \pm3\ \pm9}{\pm1\ \pm2}\qquad \pm1,\ \pm\frac{1}{2},\ \pm3,\ \pm\frac{3}{2},\ \pm9,\ \pm\frac{9}{2}$$

Factor Theorem

If a polynomial is divisible by $(x-k)$
then $p(k)=0$ remainder has to be 0.

Is $x-1$ a factor of $2x^4+3x^2-5x+7$?

$$\begin{array}{r|rrrrr} 1 & 2 & 0 & 3 & -5 & 7 \\ & & 2 & 2 & 5 & 0 \\ \hline & 2 & 2 & 5 & 0 & 7 \end{array}$$

$x-1$ is not a factor

Is $x+4$ a factor of $5x^4+16x^3-15x^2+8x+16$?

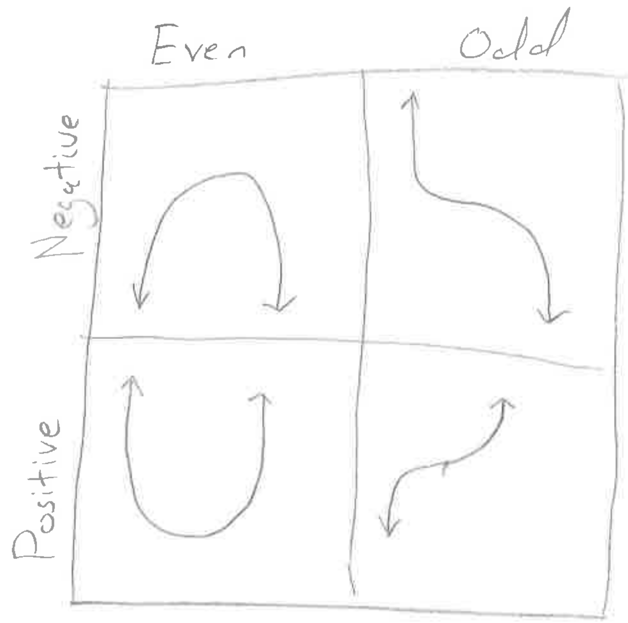
$$\begin{array}{r|rrrrr} -4 & 5 & 16 & -15 & 8 & 16 \\ & & -20 & 16 & -4 & -16 \\ \hline & 5 & -4 & 1 & 4 & 0 \end{array}$$

$x+4$ is a factor

End Behavior

If the leading coefficient is odd, the end behavior is ~~up down~~ up down or down up

If the sign is negative, the



If it is even, both sides are in the same direction

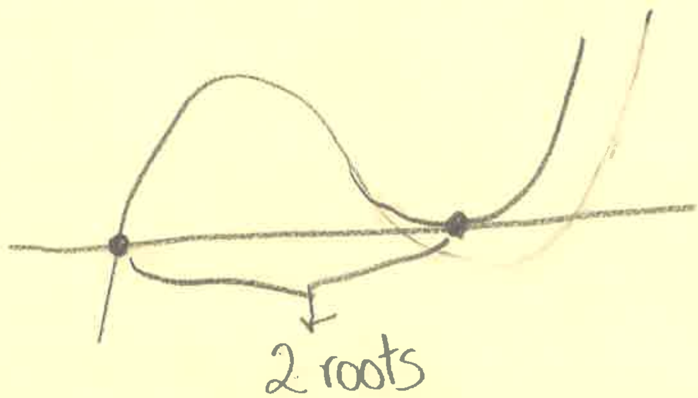
Factoring

$$
\begin{array}{r}
x + 5 \\
(x-1)\overline{)\ \cancel{x}^2 + 4x + 22} \\
-\ \cancel{x}^2 - x \\
\hline
5x + 22 \\
-\ 5x - 5 \\
\hline
\underline{27}
\end{array}
$$

If the remainder is 0, the quotient is a root

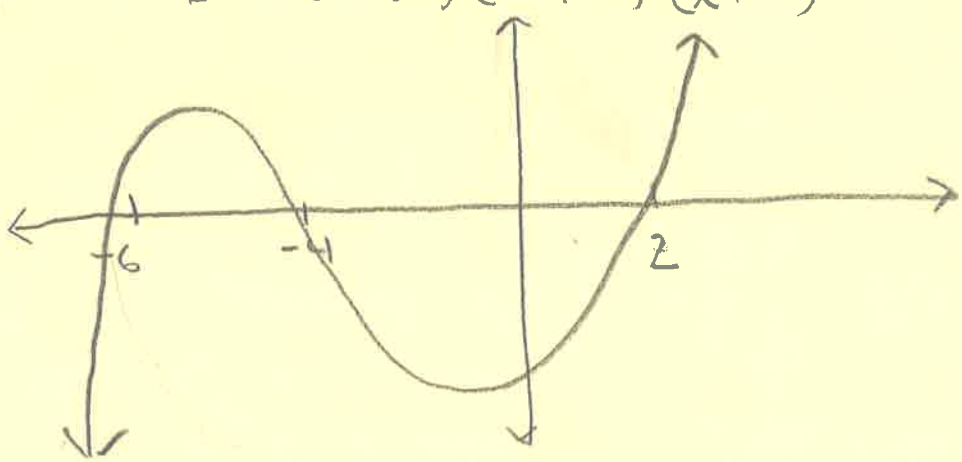
Multiplicity

- Multiplicity of a member of a multiset is the number of times it appears in the multiset



Graphing

Ex: $(x-2)(x+4)(x+6)$



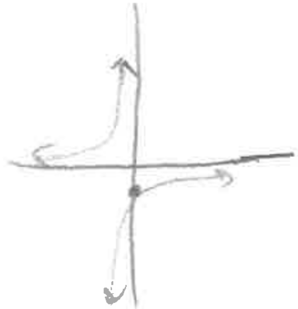
Rational Functions

y-Intercept

$$y = \frac{x^2 + x - 3}{x + 4}$$

$$y = \frac{0^2 + 0 - 3}{0 + 4}$$

$$y = \frac{-3}{4}$$



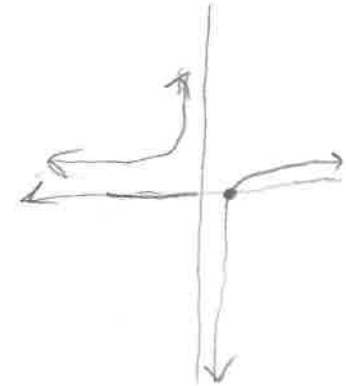
X-intercept

$$y = \frac{x - 3}{x + 4}$$

$$(x+4)\,0 = \frac{x - 3}{x + 4}\,(x+4)$$

$$0 = x - 3$$

$$x = 3$$



Solving Rational Equations

$$\frac{2x - 5}{x - 2} = \frac{11}{x - 4}$$

$$2x^2 - 5x - 8x + 20 = 11x - 22$$

$$2x^2 - 13x + 20 = 11x - 22$$
$$+22 \qquad +22$$

$$2x^2 - 13x + 42 = 11x$$
$$-11x \qquad -11x$$

$$2x^2 - 24x + 42 = 0$$

$$x = 6 - \sqrt{15}$$
$$x = 6 + \sqrt{15}$$

10 # TRIGONOMETRY 12

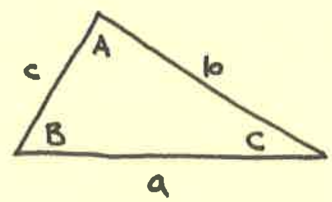
LOC Law of cosines

In any ΔABC

$$a^2 = b^2 + c^2 - 2bc\cos A$$

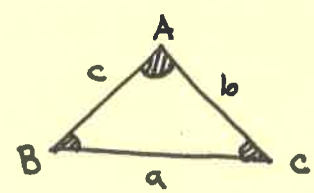
$$\text{or } b^2 = a^2 + c^2 - 2ac\cos B$$

$$\text{or } c^2 = a^2 + b^2 - 2ab\cos C$$



LOS Law of sines

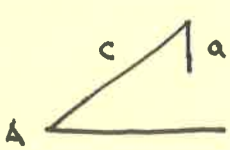
In any triangle ABC with sides a, b, c units in length, and opposite angles A, B, C respectively



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad \text{or} \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin A}$$

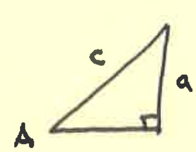
~~AMBIGUOUS~~ case (SSA)

$a < h$



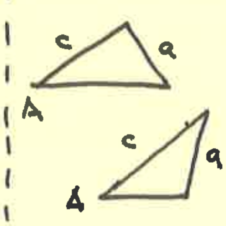
NO possible triangle

$a = h$



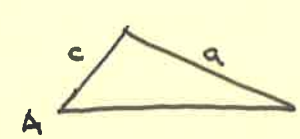
1 triangle can be formed

$h < a < c$



2 triangles possible

$a \geq c$



1 triangle is possible

Sampriktа Basu

CHAPTER 7/9

Geometric Sequences

Equations

$$U_n = U_2 r^{n-1} \qquad S_n = \frac{U_1(1-r^n)}{1-r} \quad r \neq 1 \qquad S_\infty = \frac{U_1}{1-r} \quad |r|<1$$

Examples:

The first three terms of a geometric sequence are,
$U_1 = 0.8 \quad U_2 = 2.4 \quad U_3 = 7.2$

a) Find the value of r

$$\frac{U_2}{U_1} = r \qquad r = \frac{2.4}{0.8} \qquad \boxed{r = 3}$$

b) Find the value of S_8

$$S_n = \frac{U_1(1-r^n)}{1-r} \qquad S_8 = \frac{0.8(1-3^8)}{1-3} \qquad S_8 = 1749.6$$

c) Find the least value of n such that $S_n > 35{,}000$

$$S_n = \frac{U_1(1-r^n)}{1-r} \qquad 35000 = \frac{0.8(1-3^n)}{1-3} \qquad -87501 = -3^n$$

$$\boxed{n = 10.36}$$

Sigma Notation

Go to this value

$$\sum_{n=1}^{4} 3(0.9)^{n-1}$$

What to sum

Start at this value

Akhil Pothineni

CHAPTER 7/9

Induction - Series

By Mathematical induction, prove that $\sum_{i=1}^{n} i(i+2) = \frac{n(n+1)(2n+7)}{6}$ for all integers n, $n \geq 1$.

Step 1: If $n=1$ $(1)(1+2) = 3$ ✓

Step 2: If $n=k$ Assume $\sum_{i=1}^{k} i(i+2) = \frac{k(k+1)(2k+7)}{6}$

is true for $n=k$ $k \in Z^+$

Step 3: If $n=k+1$ $\sum_{i=1}^{k} i(i+2) + (k+1)(k+1+2)$

$$= \frac{k(k+1)(2k+7)}{6} + (k+1)(k+3)$$

$$= \frac{k(k+1)(2k+7) + 6(k+1)(k+3)}{6}$$

$$= \left(\frac{k+1}{6}\right)\left(k[2k+7] + [6 \cdot (k+3)]\right)$$

$$= \frac{k+1}{6}\left(2k^2 + 13k + 18\right)$$

$$= \left(\frac{k+1}{6}\right)(k+2)(2k+9)$$

$$= \frac{(k+1)(k+2)(2(k+1)+7)}{6}$$

The conjecture is true for $k+1$

$\therefore \sum_{i=1}^{n} i(i+2) = \frac{n(n+1)(2n+7)}{6}$ is true for $n \geq 1$

Anushka Sarode

Blake Abrahamsen

Chapter 7/9

Summation of an Arithmetic Sequence:

$$\sum_{n=1}^{n} u_1 + d(n-1)$$

ending term → n; $n=1$ ← starting term; 1st term → u_1; common difference → d; end term → -1

$-6, 1, 8, 15 \ldots 71$

1.) Fill in the equation $u_n = u_1 + (n-1)d$

$u_1 = -6$

$d = 15 - 8$ or $8 - 1$ etc.

$d = 7$

$u_n = -6 + (n-1)7$

2.) Solve for the nth value of the final.

$71 = -6 + (n-1)7$
$+6 \quad +6$

$\frac{77}{7} = \frac{(n-1)7}{7}$

$11 = n-1$
$+1 \quad +1$

$n = 12$

3.) Use the summation equation:

$S_n = \frac{n}{2}(u_1 + u_n)$

$S_{12} = \frac{12}{2}(-6 + 71)$

$S_{12} = 390$

Ex.

$4, 23, 42, 61, \ldots 498$

$u_n = u_1 + (n-1)d$

$u_n = 4 + (n-1)19$

$498 = 4 + (n-1)14$
$-4 \quad -4$

$\frac{494}{14} = \frac{(n-1)14}{14}$

$26 = n-1$
$+1 \quad +1$

$n = 27$

$S_n = \frac{n}{2}(u_n + u_1)$

$S_{27} = \frac{27}{2}(4 + 498)$

$S_{27} = 6,777$

Chapter 7 and 9

Use mathematical induction to prove that $n^3 - n + 3$ is divisible by 3 for $n \in Z^+$

$n^3 - n + 3 = p(n)$

1) If $p(1) \Rightarrow 1^3 - 1 + 3 = 3A$

$3 = 3A$

$P(1)$ is true

2) If $n = k$ $k^3 - k + 3 = 3A$

$k^3 + 3 = 3A + k$

assume $p(k)$ is true

3) If $n = k+1$ $(k+1)^3 - (k+1) + 3$

$k^3 + 3k^2 + 3k + 1 - k - 1 + 3$

$3A + k + 3k^2 + 2k$

$3A + 3k^2 + 3$

$3(A + k^2 + k)$

4) $\therefore$ therefore $p(n)$ is divisible by 3 for $n \in Z^+$

Gabriel R.

Amanda zeng

Chapter 13

trigonometric equations

compound angles:

$\sin(A \pm B) = \sin A \cos A \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

ex: find the value of $\tan\left(\frac{\pi}{12}\right)$ in simplest form.

$\tan\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right)$

$\dfrac{\tan\frac{\pi}{3} - \tan\frac{\pi}{4}}{1 + \tan\frac{\pi}{3}\tan\frac{\pi}{4}}$

$\dfrac{\sqrt{3} - 1}{1 + \sqrt{3}}\left(\dfrac{\sqrt{3}-1}{\sqrt{3}-1}\right)$

$\dfrac{3 - 2\sqrt{3} + 1}{3 - 1} = \dfrac{4 - 2\sqrt{3}}{2}$

$= \boxed{2 - \sqrt{3}}$

Double Angle Identities:

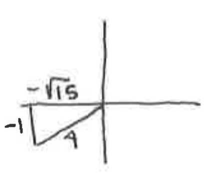
$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta \;/\; 1 - 2\sin^2\theta$

$\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

ex. The angle θ lies in the 3rd quadrant and $\sin\theta = -\frac{1}{4}$

Find $\sin 2\theta$.



$\sin 2\theta = 2\left(-\frac{1}{4}\right)\left(-\frac{\sqrt{15}}{4}\right)$

$= 2\left(+\frac{\sqrt{15}}{16}\right)$

$= +\dfrac{\cancel{2}\sqrt{15}}{\cancel{16}} = \boxed{+\dfrac{\sqrt{15}}{8}}$

Declan Kelly

Chapter 13

arcsine, arccosine, arctangent

$\arcsin(x) = \sin^{-1}(x)$ $\arccos(x) = \cos^{-1}(x)$

$\arctan(x) = \tan^{-1}(x)$

so if

$\sin(30^\circ) = \frac{1}{2}$ ~~th~~ than $\arcsin(\frac{1}{2}) = 30^\circ$

example:

without using tech, show that

$$\arctan\left(\frac{1}{6}\right) + \arctan\left(\frac{5}{7}\right) = \frac{\pi}{4}$$

$$\tan\left(\arctan\left(\frac{1}{6}\right) + \arctan\left(\frac{5}{7}\right)\right) = \tan\frac{\pi}{4}$$

$$\frac{\tan(\arctan(\frac{1}{6})) + \tan(\arctan(\frac{5}{7}))}{1 - \tan(\arctan(\frac{1}{6})) \cdot \tan(\arctan(\frac{5}{7}))} = \tan\left(\frac{\pi}{4}\right)$$

$$\frac{\frac{1}{6} + \frac{5}{7}}{1 - \frac{1}{6} \cdot \frac{5}{7}} = 1$$

$$\frac{\frac{7}{42} + \frac{30}{42}}{1 - \frac{5}{42}} = 1$$

$$\frac{\frac{37}{42}}{\frac{37}{42}} = 1$$

$$\boxed{1 = 1}$$

Chapter 13.

Kevin Li

Identity Proof

☆ $\tan\theta = \frac{\sin\theta}{\cos\theta}$ ☆ $\cos^2\theta + \sin^2\theta = 1$

☆ $\sec\theta = \frac{1}{\cos\theta}$ ☆ $1 + \tan^2\theta = \sec^2\theta$

☆ $\text{Cosec}\,\theta = \frac{1}{\sin\theta}$ ☆ $1 + \cot^2\theta = \csc^2\theta$

☆ $\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

☆ $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

☆ $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$ ☆ $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

☆ $\sin 2\theta = 2\sin\theta\cos\theta$

☆ $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

1. Verify: $\cos^4 x - \sin^4 x = 1 - 2\sin^2 x$

$(\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x)$

$(\cos^2 x - \sin^2 x)(1 \cancel{- \sin^2 x \mp \sin^2 x})$

$(\cos^2 x - \sin^2 x)$

$(1 - \sin^2 x - \sin^2 x) \longrightarrow 1 - 2\sin^2 x$

2. Verify $\sin x + \cot x \cos x = \csc x$

$\sin x \cdot \frac{\sin x}{\sin x} + \frac{\cos x}{\sin x} \cdot \cos x =$

$\frac{\sin^2 x}{\sin x} + \frac{\cos^2 x}{\sin x} =$

$\frac{\sin^2 x + \cos^2 x}{\sin x} = \frac{1}{\sin x} = \csc x$

charith

Trig Identities

Identities*

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sec\theta = \frac{1}{\cos\theta}$

$\mathrm{cosec}\,\theta = \frac{1}{\sin\theta}$

Pythagoreean Identities*

$\cos^2\theta + \sin^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

* These identities only work for right triangles

Example) $\tan^2\theta + 3\sec\theta = 9$

$\sec^2\theta - 1 + 3\sec\theta = 9$

$(\sec\theta)^2 + 3\sec\theta - 10 = 0$

$(\sec\theta + 5)(\sec\theta - 2) = 0$

$\boxed{\sec\theta = 2 \quad \sec\theta = -5}$

Identities Used:

$1 + \tan^2\theta = \sec^2\theta$

Counting: Chapter 8

Brian, Trihan, Jayce, Pranav

Combinations: nCr or $\binom{n}{r}$: number of possible subsets of length r from n. Order of items does not matter. Equal to $\boxed{\frac{n!}{(n-r)!\,r!}}$

Permutations: nPr: like combinations, but order matters. Equal to $\boxed{\frac{n!}{(n-r)!}}$

Example:

Skyline has a Math contests and 20 students enter. How many ways to win 3 gold medals and 5 silver medals?

answer

$$\binom{20}{3}\binom{17}{5} = 7054320$$

\# of students; \# of students after gold medals; \# of gold meds; \# of silver medals

How many ways to seat 20 students in math contest?

$20! \approx 2.43 \cdot 10^{18}$

\# of students

$$\binom{20}{8}\binom{8}{5} = 7054320$$

\# of winners; \# of silver medals

The following year, they decide medals are too much trouble and do 1–5th place instead. How many ways to fill these places?

Answer: ${}_{20}P_5 = 1860480$

Binomial Theorem

$$\boxed{(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k}$$

example:

$(x+y)^2 = \binom{2}{0}x^2 + \binom{2}{1}xy + \binom{2}{2}y^2 = x^2 + 2xy + y^2$

$(x+y)^3 = \binom{3}{0}x^3 + \binom{3}{1}x^2y + \binom{3}{2}xy^2 + \binom{3}{3}y^3$

$= x^3 + 3x^2y + 3xy^2 + y^3$

Find k given that the constant term of $\left(\sqrt{x} + \frac{k}{x}\right)^9$ is -4032.

$\frac{1}{2}(9-r) - r = 0 \qquad r = 3$

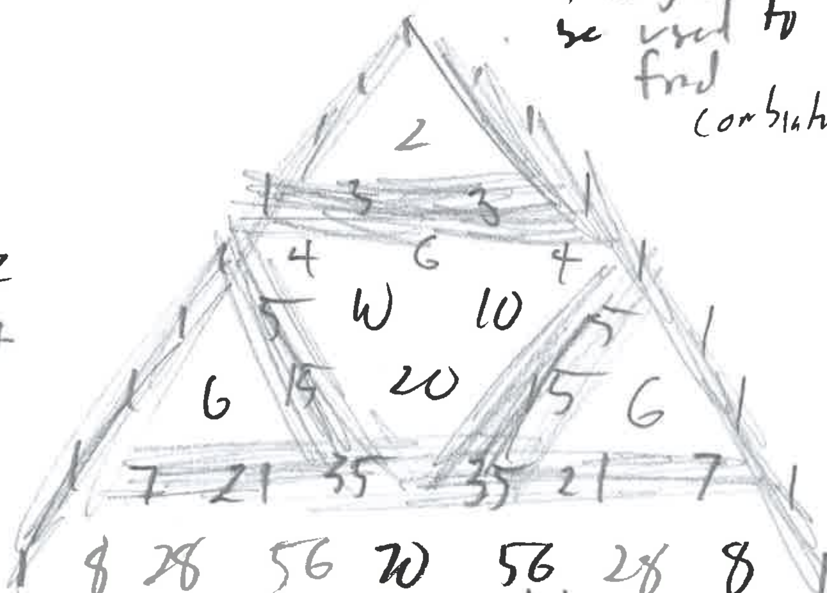
$\binom{9}{3}(\sqrt{x})^6\left(\frac{k}{x}\right)^3$

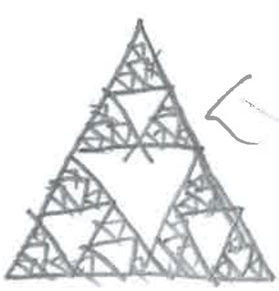
$84k^3 = -4032$

$k^3 = -\frac{4032}{84}$

$k = \sqrt[3]{-48}$

Pascal's triangle can be used to find combinations

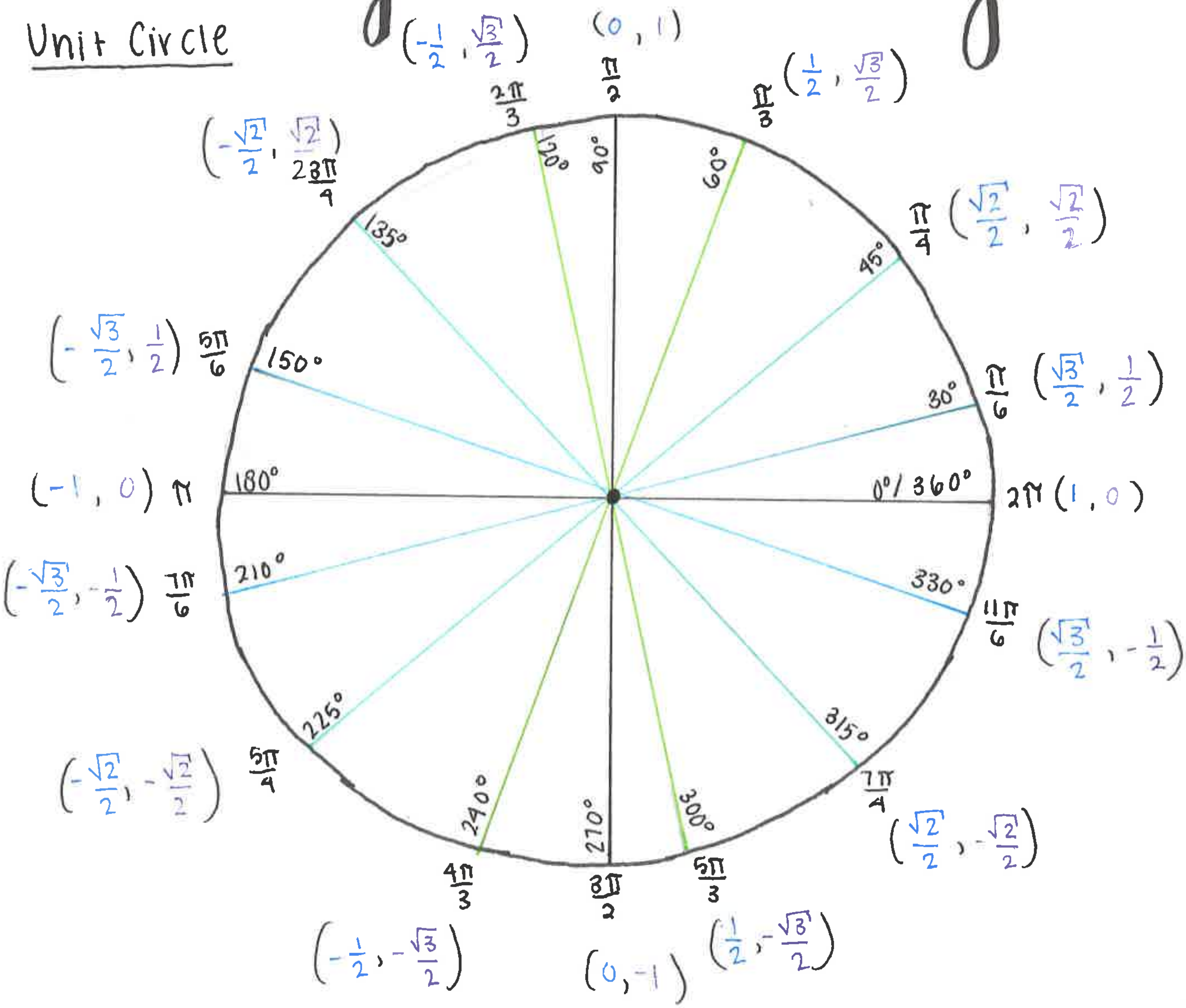




If you shade the odd numbers on Pascal's triangle, you will get the Sierpenski triangle.

Trigonometry

Unit Circle



6 Trig Functions

$\sin = \frac{opp}{hyp}$

$\cos = \frac{adj}{hyp}$

$\tan = \frac{opp}{adj}$

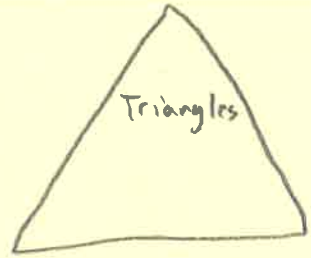
$\csc = \frac{1}{\sin} = \frac{hyp}{opp}$

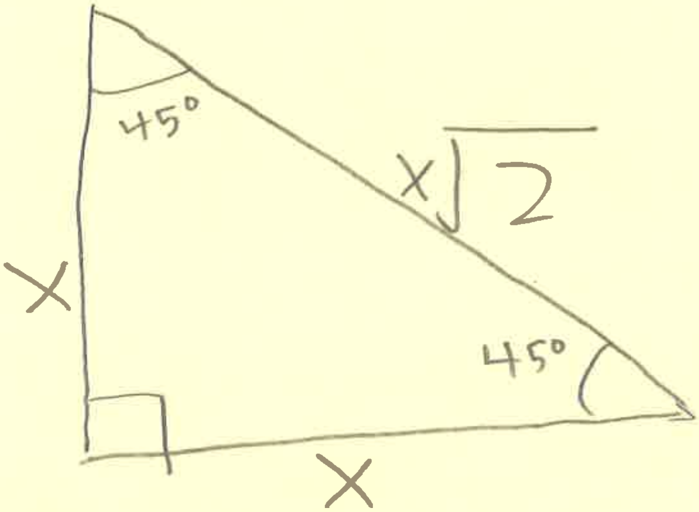
$\sec = \frac{1}{\cos} = \frac{hyp}{adj}$

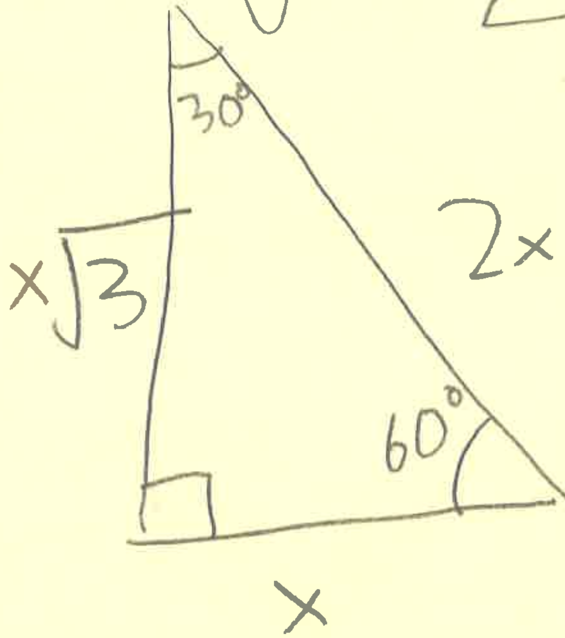
$\cot = \frac{1}{\tan} = \frac{adj}{opp}$

Zoe Walschots

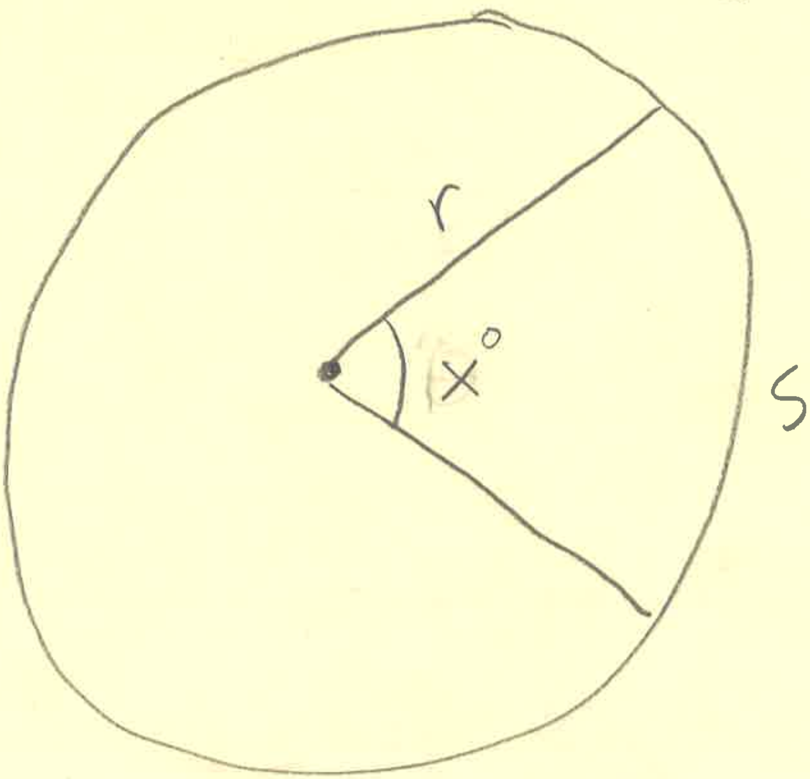
Special Right







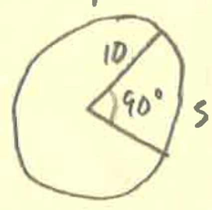
Arc Length



Equation :

$$s = 2\pi r \cdot \frac{x}{360}$$

Example :



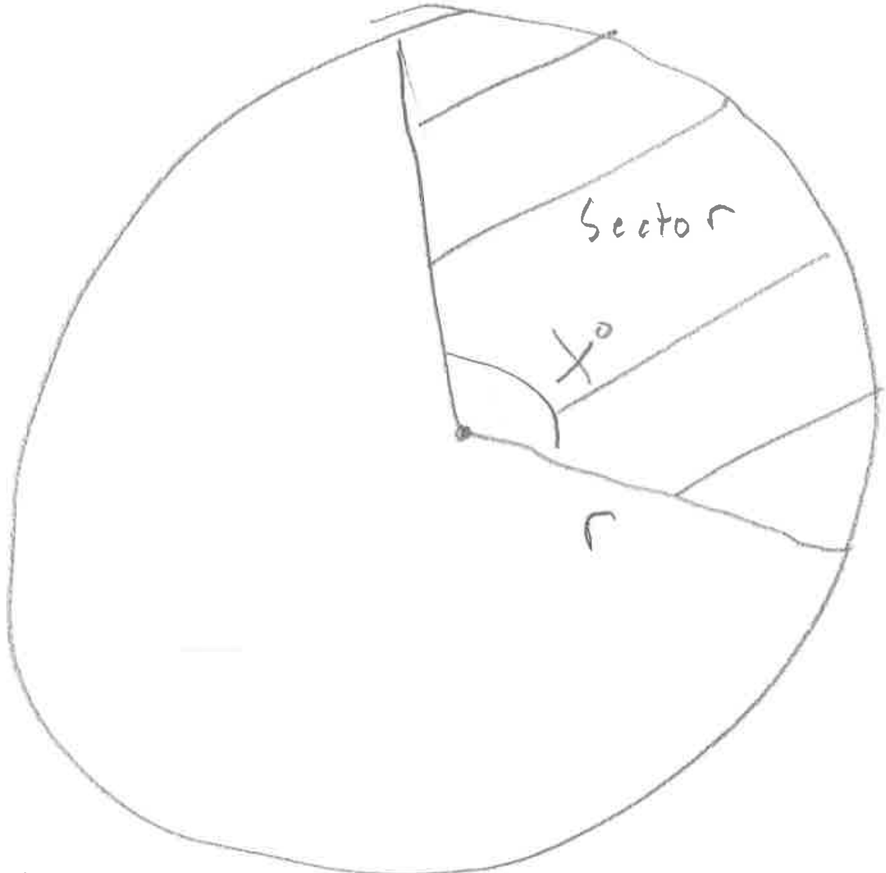
$s = 2\pi \cdot 10 \cdot \frac{90}{360}$

↓

$\boxed{s = 5\pi \text{ units}}$

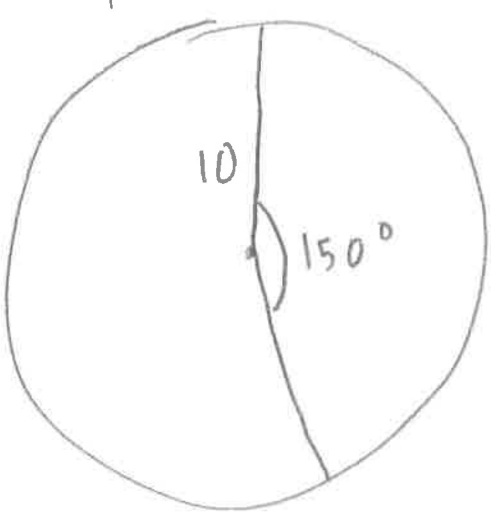
Kevin Xu

Sector Area



Sector Area: $\pi r^2 \cdot \frac{x}{360°}$

Example:



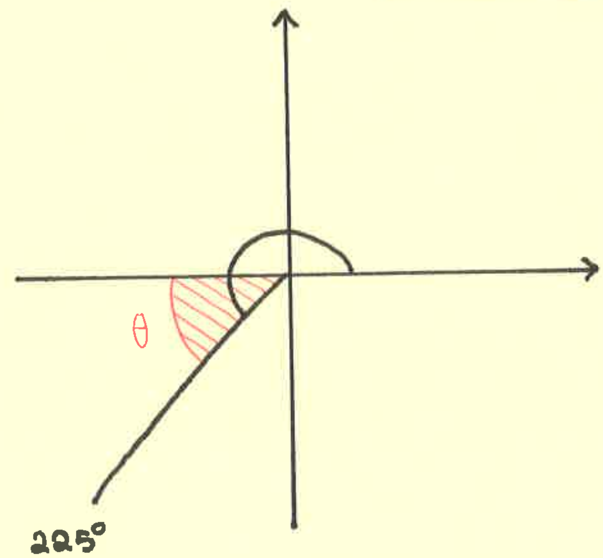
Sector $= 10^2 \cdot \pi \cdot \frac{150}{360}$

$\boxed{\text{Sector} = \frac{125}{3}\pi \text{ units}^2}$

Kevin Xu

Reference Angles

↳ the angle that the line makes with the x-axis that is less than or equal to 90°.



Eg: Looking at the picture, find the reference angle, θ.

$\theta = [\text{Original Angle}] - 180^\circ$

$\theta = 225^\circ - 180^\circ$

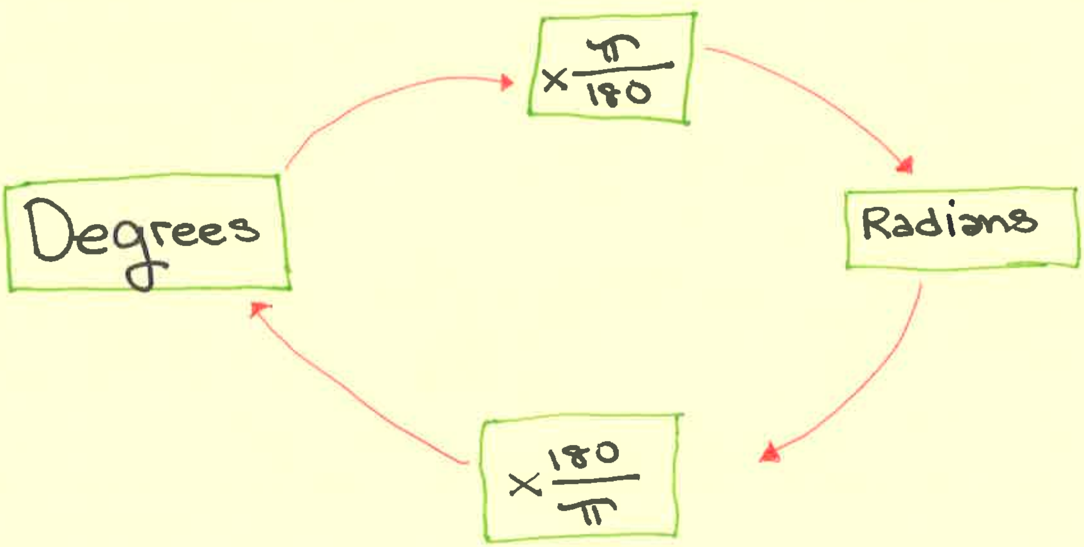
$\boxed{\theta = 45^\circ}$

Degrees & Radians

One full revolution = 360° OR 2π

Half a revolution = 180° OR π

- Degrees-Radians Conversions



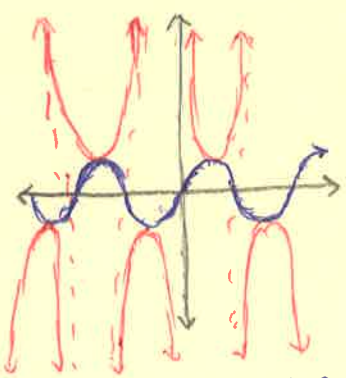
Ch 10-12: Trigonometry

Daniel
Hong

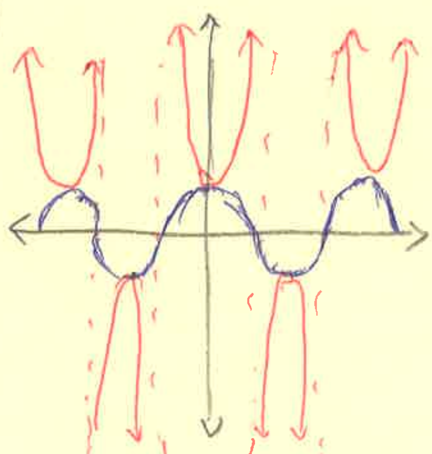
Graphs and Transformations

Graphing:

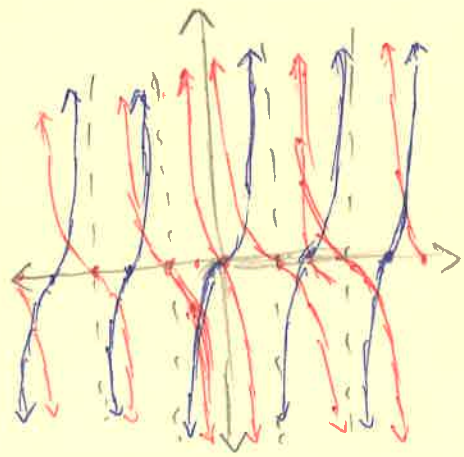
Parent functions and their graphs:



$y = \sin(x)$: period 2π
$y = \csc(x)$: period 2π



$y = \cos(x)$: period 2π
$y = \sec(x)$: period 2π



$y = \tan(x)$: period π
$y = \cot(x)$: period π

Transformations:

General form of trigonometry function

$$\boxed{y = a\sin[b(x-c)] + d}$$

can be replaced with any 6 of the parent trig functions, transformations will be the same

$|a|$ = amplitude ⟶ changing a will vertically dilate

$\frac{2\pi}{b}$ = period ⟶ increasing b horizontally shrinks, decreasing will stretch

c = horizontal translation

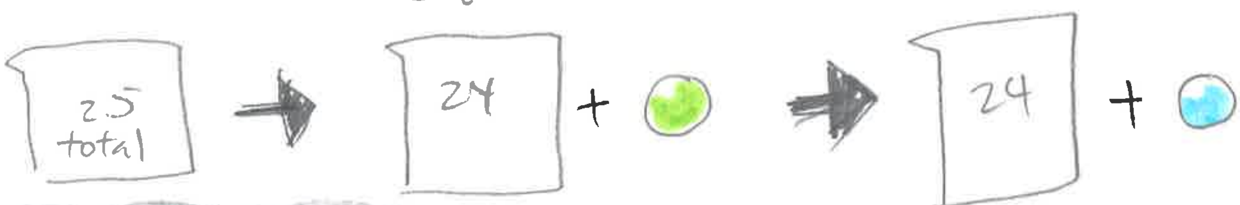
d = midline ⟶ increasing d translates graph up, decreasing d translates graph down

Probability: (24)

Dependency

① Independent = with replacement

↳ Draw one, put it back (replaces), and draw another one.



② Dependent = Without Replacement

↳ Draw consecutively, without replacement.

• Conditional Probability

- P(A) given that P(B) happened

↳ Also, P(A different marble) given that the first marble hasn't been replaced

$$\boxed{P(A \mid B)} = \frac{P(A \cap B)}{P(B)}$$

Chapter 24. Probability

Experimental: What actually happens, the results obtained

Experimental Probability = Relative Frequency

- # of trials : times repeated
- Outcomes : different possible results
- frequency : # of particular outcome
- relative frequency : frequency of particular outcome

Theoretical : What is expected to happen

$$\frac{\text{\# of favorable outcomes}}{\text{\# of possible outcomes}}$$

Relative Frequency : $\dfrac{\text{\# of times an event occurs}}{\text{total \# of trials}}$

Conditional Probability :

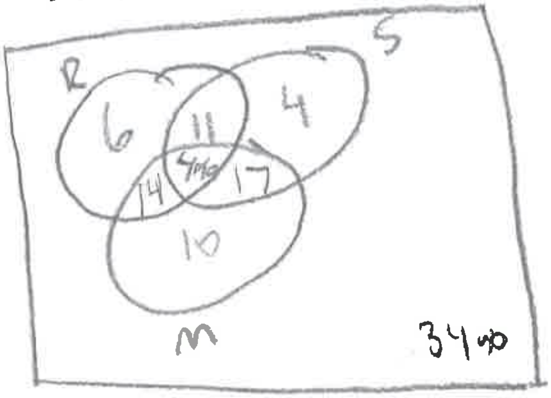
- $P(A|B) \rightarrow$ Probability A Given Probability B
- $\dfrac{P(A \cap B)}{P(B)}$

Trishan A

Venn Diagrams &
Tree Diagrams

Venn Students were surveyed about what types of club events they attended.
25% - robotics, 45% - math, 36% - science, 14% - rob & math, 21% math & science, 15 - rob & science, 4%: all 3

a.



b. Find the probability that the person attended none of these events

$\boxed{34\%}$

c. Find the probability that a person attends robotics & science competitions only

$\boxed{11\%}$

Mutually Exclusive: Events that cannot occur at the same time; are disjoint.

$P(A \cap B) = \emptyset$

Empty Set: An Empty set is a set without elements, size of it is zero.

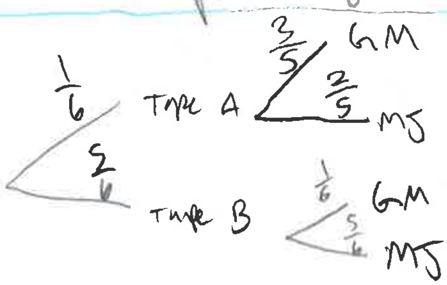
Complementary: Opposite of what is given.

$P(A')$ = Probability of not A

$P(A \cap B)'$ = Prob of not A & B

Tree Mrs. Shim goes disco dancing where the songs are played from tape A or B. If 1 is rolled, tape A is played. If 2,3,4,5,6 is rolled tape B is played. Tape A: 3 songs by George Michael & 2 songs by MJ. Tape B: 1 song by George Michael & 5 songs by MJ.

What is the probability that Michael Jackson is played.



$$\frac{1}{6}\cdot\frac{2}{5}+\frac{5}{6}\cdot\frac{5}{6}=\frac{2}{30}+\frac{25}{36}=\boxed{\frac{137}{180}}$$