Examples of Modulus Inequalities

1) $\frac{2x}{|x-1|} < 1$ $\quad (x \neq 1)$

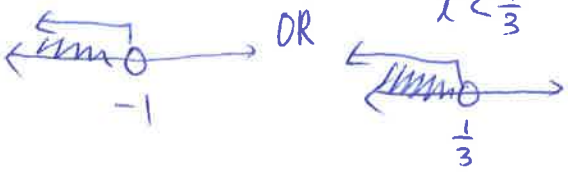
$\hookrightarrow 2x < |x-1|$

OR

① $x-1 > 2x$ ② $x-1 < -2x$

$-1 > x$ $\qquad 3x < 1$

$x < \frac{1}{3}$



$\therefore x \in (-\infty, \frac{1}{3})$

2) $\frac{2x}{|x-1|} > 3$

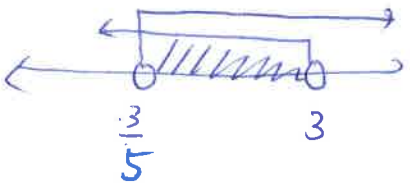
$\hookrightarrow 2x > 3|x-1|$

$\Rightarrow |x-1| < \frac{2x}{3}$

① $x-1 < \frac{2x}{3}$ and $x-1 > \frac{-2x}{3}$

$\frac{1}{3}x < 1$ $\qquad \frac{5}{3}x > 1$

$x < 3$ and $x > \frac{3}{5}$



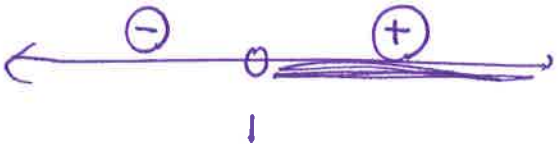
$$\left|\frac{2x+3}{x-1}\right| \geq 2$$

$$\frac{2x+3}{x-1} \geq 2 \quad \text{OR} \quad \frac{2x+3}{x-1} \leq -2$$

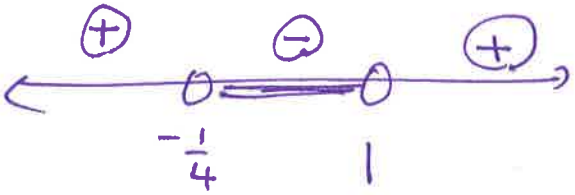
$$\frac{2x+3}{x-1} - \frac{2(x-1)}{x-1} \geq 0 \qquad \frac{2x+3}{x-1} + \frac{2(x-1)}{x-1} \leq 0$$

$$\frac{2x+3-2x+2}{x-1} \geq 0 \qquad \frac{2x+3+2x-2}{x-1} \leq 0$$

$$\frac{5}{x-1} \geq 0 \qquad \frac{4x+1}{x-1} \leq 0$$



$x > 1$



$$\Rightarrow \boxed{x \in \left(-\frac{1}{4}, 1\right) \cup (1, \infty)}$$

$$y = \frac{3x-2}{2x-1}.$$

$$= \frac{3}{2} + \frac{-\frac{1}{2}}{2x-1}.$$

$$= \frac{3}{2} + \frac{-\frac{1}{2}}{2(x-\frac{1}{2})}$$

$$\begin{array}{r|l} & \frac{3}{2} \\ \hline 2x-1 & 3x-2 \\ - & 3x-\frac{3}{2} \\ \hline & -\frac{1}{2} \end{array} \qquad \begin{array}{l} -2 \\ +\frac{3}{2} \end{array}$$

① U.D BAFO $\frac{1}{2}$

② H.D BAFO $\frac{1}{2}$.

③ Reflect over x-axis

④ V.T: up $\frac{3}{2}$

⑤ H.T: Right $\frac{1}{2}$.

as $x \to \infty$

$y \to \frac{3}{2}$.

as $x \to -\infty$

$y \to \frac{3}{2}$.