

- Examples) a) Write the rational function in the form of $y = q(x) + \frac{r(x)}{d(x)}$
 b) Describe the transformations. ($y = \frac{1}{x}$)
 c) Find the horizontal asymptote.
 d) Finally state the end behavior of the graph as x approaches to positive and negative infinity.

a) $y = \frac{2x+1}{4x^2-4x-3}$
 $y = \frac{2x+1}{(2x+1)(2x-3)}$
 $= \frac{1}{2x-3}$

V.A: $x = \frac{3}{2}$
 Hole: $x = -\frac{1}{2}, y = \frac{1}{4}$
 x-int: None
 y-int: $(0, \frac{1}{3})$

b) $y = \frac{x^2+3x-4}{2x^2+7x-4}$

$y = \frac{(x+4)(x-1)}{(2x-1)(x+4)}$
 $= \frac{x-1}{2x-1}$

V.A: $x = \frac{1}{2}$
 Hole: $x = -4$
 $y = \frac{-4-1}{(2)(-4)-1}$
 $= \frac{-5}{-9} = \frac{5}{9}$
 $(-4, \frac{5}{9})$
 X-int: $x-1=0$
 $x=1$
 $(1, 0)$
 y-int: $(0, 1)$

a) $y = \frac{0}{2x-3} + \frac{1}{2x-3}$

b) $y = \frac{1}{2(x-\frac{3}{2})}$
 V.D: None
 H.D: BOFO: $\frac{1}{2}$
 V.T: None
 H.T: $\frac{3}{2}$ Right

c) Horizontal Asymptote: $y=0$

d) as $x \rightarrow \infty \Rightarrow y \rightarrow 0$
 $x \rightarrow -\infty \Rightarrow y \rightarrow 0$

a) $2x-1 \overline{) x-1}$
 $-) x-\frac{1}{2}$
 $\hline \frac{1}{2}$
 $\Rightarrow y = \frac{1}{2} + \frac{\frac{1}{2}}{2x-1}$

b) $y = \frac{1}{2} + \frac{\frac{1}{2}}{2(x-\frac{1}{2})}$
 V.D: BOFO: $\frac{1}{2}$
 Reflect over X-axis
 H.D: BAFO: $\frac{1}{2}$
 V.T: up $\frac{1}{2}$
 H.T: Right $\frac{1}{2}$

c) H.A: $y = \frac{1}{2}$

d) as $x \rightarrow \infty, y \rightarrow \frac{1}{2}$
 as $x \rightarrow -\infty, y \rightarrow \frac{1}{2}$