

Example) Find the horizontal asymptotes by performing a long or synthetic division. Then state the end behavior of the graph as  $x$  approaches infinity.

a)  $y = \frac{0x^2 + 2x + 1}{4x^2 - 4x - 3}$

Handwritten work shows synthetic division of  $2x + 1$  by  $4x^2 - 4x - 3$ . The result is  $\frac{1}{2x-3}$ . The horizontal asymptote is  $y = 0$ .

OR  $2x \div 4x^2 = \frac{1}{2}$

$\rightarrow$  H.A :  $y = 0$

b)  $y = \frac{1x^2 + 3x - 4}{2x^2 + 7x - 4}$

Handwritten work shows synthetic division of  $1x^2 + 3x - 4$  by  $2x^2 + 7x - 4$ . The result is  $y = \frac{1}{2}$ .

$\rightarrow$  H.A  $\Rightarrow y = \frac{1}{2}$

As  $x \rightarrow \infty$   $y \rightarrow \frac{1}{2}$   
As  $x \rightarrow -\infty$   $y \rightarrow \frac{1}{2}$

Practice WS

Identify every value of  $y$  for which there appears to be a horizontal asymptote. Using your graphing calculator, determine whether the graph of the function has a horizontal asymptote by selecting a window with an extremely large  $x$  value in the positive and negative direction.

1.  $f(x) = \frac{5x^2 - 3x + 1}{2x^2 + 4x - 7}$

2.  $f(x) = \frac{3x^3 - x + 1}{6x^3 + 2x^2 - 7}$

3.  $f(x) = \frac{4 - 7x}{2 + 3x}$

4.  $f(x) = \frac{(3x+4)(2x+1)}{(4x+1)(x-1)}$

5.  $f(x) = \frac{2x^2 - x + 3}{x^3 + 1}$

6.  $f(x) = \frac{x^2 + 4}{3 - 4x}$

7.  $f(x) = \frac{4x}{x^2 + 9}$

8.  $f(x) = \frac{1}{x^2 + 1}$

9.  $f(x) = \frac{x^3}{x^2 + 1}$

10. Describe how you could determine horizontal asymptotes using only the algebraic representation of a function.