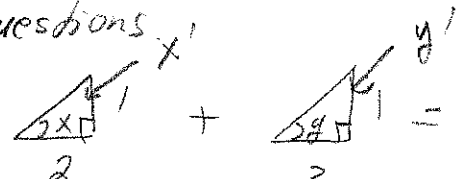


#1 (a)  $\tan x' + \tan y' = \frac{1}{2} + \frac{1}{3}$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \cdot \frac{1}{3}} = \frac{\frac{5}{6}}{\frac{5}{6}} = 1$$

$$\arctan[\tan(x+y)] = \arctan(1) = \frac{\pi}{4}$$

(b) $\arctan 2 + \arctan 3$
 $x' + y'$



$$x + x' = 90^\circ$$

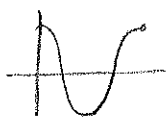
$$y + y' = 90^\circ$$

$$x + y + x' + y' = 180^\circ$$

$$x' + y' = 180^\circ - (x + y) = 180 - 45$$

$$= 135^\circ \Rightarrow \frac{5\pi}{4} \text{ rad}$$

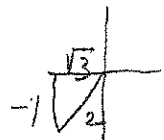
#2 (a) $x + 2 \cos x = x$



$$\cos x = 0 \quad [0, 2\pi)$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\sin x = -\frac{1}{2} \quad x = \pi + \frac{\pi}{6} = \frac{5\pi}{6}$$



$$A = \frac{\pi}{2} \Rightarrow a = 1 \quad B = \frac{5\pi}{6} \quad b = 5$$

$$C = \frac{3\pi}{2} \Rightarrow b = 3$$

(b) $0 \leq x \leq 2\pi$: Domain

$$1 \leq x + 2 \cos x \leq 2\pi + 2$$
 : Range

#3. $\cos 2\theta = 2 \cos^2 \theta - 1$ $\theta = \frac{1}{2}x$ (2)

(a) $2 \cos^2 \theta = \cos 2\theta + 1$

$$\cos^2 \theta = \frac{\cos 2\theta + 1}{2}$$

$$\cos \theta = \sqrt{\frac{\cos 2\theta + 1}{2}}$$

$$\Rightarrow \sqrt{\frac{\cos x + 1}{2}} = \cos \frac{x}{2}$$

$$0 \leq x \leq \pi$$

$$0 \leq \frac{x}{2} \leq \frac{\pi}{2}$$

(b) $\cos 2\theta = 1 - 2 \sin^2 \theta$

$$2 \sin^2 \theta = 1 - \cos 2\theta$$

$$\sin \theta = \sqrt{\frac{1 - \cos 2\theta}{2}}$$

$$\Rightarrow \sin \frac{x}{2} = \sqrt{\frac{1 - \cos x}{2}}$$

#4. $\cos 4x = \cos (2x + 2x)$

$$= \cos 2x \cdot \cos 2x - \sin 2x \sin 2x$$

$$= \cos^2 2x - \sin^2 2x$$

$$= 1 - \sin^2 2x - \sin^2 2x = 1 - 2 \sin^2 2x = 1 - 2 \left(\frac{-5}{9}\right)^2$$

$$= 1 - 2 \left(\frac{25}{81}\right)$$

$$= 1 - \frac{50}{81} = \frac{81}{81} - \frac{50}{81} = \frac{31}{81}$$

Given $(\sin x + \cos x)^2 = \left(\frac{2}{3}\right)^2$

$$\sin^2 x + 2 \sin x \cos x + \cos^2 x = \frac{4}{9}$$

$$2 \sin x \cos x = \frac{4}{9} - 1 = \frac{-5}{9} = \sin 2\theta \quad (\text{Double Angle identity})$$

#5. $3(\cos^2 x - 8\cos x + 4) = 0$ (5)

(a)
$$\begin{array}{r} 3 \cos x \quad -2 \\ 1 \cos x \quad -2 \end{array}$$

$= (3\cos x - 2)(\cos x - 2) = 0$

$\cos x = \frac{2}{3}$ ~~$\cos x = 2$~~

$x = \cos^{-1}\left(\frac{2}{3}\right)$ in degree mode since $0 \leq x \leq 180^\circ$



(b) $3\sec^2 x - 8\sec^2 x + 4 = 0$

$(3\sec^2 x - 2)(\sec^2 x - 2) = 0$

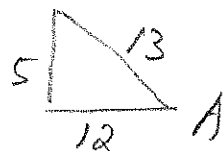
$\sec x = \pm \sqrt{\frac{2}{3}}$ $\sec x = \pm \sqrt{2}$

~~$\cos x = \pm \sqrt{\frac{3}{2}}$~~

$\cos x = \pm \frac{1}{\sqrt{2}}$

$x = 45^\circ, 135^\circ$

#6. $\sin A = \frac{5}{13}$



$$\begin{array}{r} 169 \\ - 25 \\ \hline 144 \end{array}$$

$\cos A = \frac{12}{13}$

$\Rightarrow \cos 2A = 1 - 2\sin^2 A = 1 - 2\left(\frac{5}{13}\right)^2 = \frac{169}{169} - \frac{50}{169} = \frac{119}{169}$

#7. $p(x) = 210 \sin[0.5(x - 5.2)] + 990$

x is months ($x = 1$ Jan. 2014)

(a) $p(5) = 210 \sin[0.5(5) - 2.6] + 990 =$

#7 (b) $BIS = 210 \sin[0.5t - 2.6] + 990$.

(4)

#8. $\cos\left(\frac{\pi}{6} + x\right)$



(a) (i) $= \cos\left(\frac{\pi}{6}\right)\cos x - \sin\left(\frac{\pi}{6}\right)\sin x$

$= \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x$

(ii) $\frac{\sqrt{3}\cos x - \sin x}{2} = \frac{1}{2} \cos\left(\frac{\pi}{6} + x\right) = \frac{1}{2}$

$\frac{\pi}{6} + x = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}, \frac{5\pi}{3}$

$x = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$

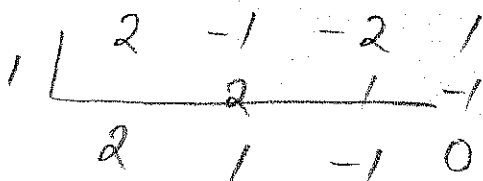
$x = \frac{5\pi}{3} - \frac{\pi}{6} = \frac{9\pi}{6} = \frac{3\pi}{2}$

#8 (b) $p(x) = 2x^3 - x^2 - 2x + 1$

(i) $p(1) = 2 - 1 - 2 + 1 = 0 \Rightarrow \therefore x = 1$ is a root.

(ii) $2x(x^2 - 1) - (x^2 - 1)$

$= (2x - 1)(x + 1)(x - 1) = 0$



$x = \frac{1}{2}, x = 1, x = -1$

$$(iii) \quad \underline{\sin 2\theta} \cos \theta + \sin^2 \theta$$

$$= 2 \sin \theta \cdot \cos \theta \cdot \cos \theta + \sin^2 \theta$$

$$= 2 \sin \theta (\cos^2 \theta) + \sin^2 \theta$$

$$= 2 \sin \theta (1 - \sin^2 \theta) + \sin^2 \theta$$

$$= 2 \sin \theta - 2 \sin^3 \theta + \sin^2 \theta$$

$$= -2 \sin^3 \theta + \sin^2 \theta + 2 \sin \theta$$

$$(iv) \quad \sin 2\theta \cos \theta + \sin^2 \theta$$

$$= -2 \sin^3 \theta + \sin^2 \theta + 2 \sin \theta = 1$$

$$= 2 \sin^3 \theta - \sin^2 \theta - 2 \sin \theta + 1 = 0$$

$$\sin \theta = 1, -1, \frac{1}{2} \quad \text{from (b)(iii)}$$

$$0 \leq \theta < 2\pi, \quad \boxed{\theta = 0, 2\pi, -\pi, \frac{\pi}{6}, \frac{5\pi}{6}}$$

$$\sqrt{2/3}$$